HYBRIDIZABLE DISCONTINUOUS GALERKIN METHODS FOR CONTINUUM MECHANICS

N. C. Nguyen[1], J. Peraire[1], and B. Cockburn[2]

[1] Department of Aeronautics and Astronautics, Massachusetts Institute of Technology, 77 Massachusetts Ave, Cambridge, MA 02139, USA, cuongng@mit.edu, peraire@mit.edu
[2] School of Mathematics, University of Minnesota, Minneapolis, MN 55455, USA, cockburn@math.umn.edu

Key words: *Computational fluid dynamics, computational solid mechanics, computational electromagnetics, hybridzable discontinuous Galerkin methods.*

We present an overview of hybridizable discontinuous Galerkin methods for solving partial differential equations (PDEs) in continuum mechanics [1, 2, 3, 4]. The essential ingredients are a local Galerkin projection of the underlying PDEs at the element level onto spaces of polynomials of degree k to parametrize the numerical solution in terms of the numerical trace; a judicious choice of the numerical flux to provide stability and consistency; and a global jump condition that enforces the continuity of the numerical flux to arrive at a global weak formulation in terms of the numerical trace. The HDG methods are fully implicit, high-order accurate and endowed with several unique features which distinguish themselves from other discontinuous Galerkin methods. First, they reduce the globally coupled unknowns to the approximate trace of the solution on element boundaries, thereby leading to a significant reduction in the degrees of freedom. Second, they provide, for smooth viscous-dominated problems, approximations of all the variables which converge with the optimal order of $k+1$ in the L^2-norm. Third, they possess some superconvergence properties that allow us to define inexpensive element-by-element postprocessing procedures to compute a new approximate solution which may converge with higher order than the original solution. And fourth, they allow for a novel and systematic way for imposing boundary conditions for the total stress, viscous stress, vorticity and pressure which are not naturally associated with the weak formulation of the methods. In addition, they possess other interesting properties for specific problems. Their approximate solution can be postprocessed to yield an exactly divergence-free velocity field for incompressible flows. They do not exhibit volumetric locking for nearly incompressible solids. We provide extensive numerical results to illustrate their distinct characteristics and compare their performance with that of continuous Galerkin methods.

REFERENCES

[1] N. C. Nguyen and J. Peraire. Hybridizable Discontinuous Galerkin Methods for Partial Differential Equations in Continuum Mechanics. *Journal of Computational Physics*, 231 (2012) 5955–5988.

[2] N. C. Nguyen, J. Peraire, and B. Cockburn. High-order implicit hybridizable discontinuous Galerkin methods for acoustics and elastodynamics. *Journal of Computational Physics*, 230 (2011), 3695–3718.

[3] N. C. Nguyen, J. Peraire, and B. Cockburn. An implicit high-order hybridizable discontinuous Galerkin method for the incompressible Navier-Stokes equations. *Journal of Computational Physics*, 230 (2011), 1147–1170.

[4] N. C. Nguyen, J. Peraire, and B. Cockburn. Hybridizable discontinuous Galerkin methods for the time-harmonic Maxwells equations. *Journal of Computational Physics*, 230 (2011), 7151–7175.