

## COMPUTATIONAL ASPECTS IN THE LARGE DISPLACEMENT INELASTIC ANALYSIS OF FRAMES WITH PLASTIC UNSTRESSING

Konstantinos V. Spiliopoulos<sup>1</sup>, Ioannis Kapogiannis<sup>2</sup>

<sup>1,2</sup> Department of Civil Engineering, Institute of Structural Analysis and Antiseismic Research,  
National Technical University of Athens, Zografou Campus, 157-80 Athens, Greece,

<sup>1</sup> [kvspilio@central.ntua.gr](mailto:kvspilio@central.ntua.gr), <http://users.ntua.gr/kvspilio>

<sup>2</sup> [john\\_kapogiannis@hotmail.com](mailto:john_kapogiannis@hotmail.com)

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It is a well-known fact that the second order effects may dramatically change the behavior of structures, particularly when materials with strain hardening or softening behaviour are considered. This inelastic behavior should be combined with an incremental analysis, which must take into account a non-holonomic law, i.e. possible plastic unstressing of a region. Nevertheless, it is customary that existing software packages generally assume holonomic behaviour in order to avoid being trapped into the recurring process of loading, unloading and reloading [1]. Avoiding, however, possible plastic unstressing may cause erroneous results.

In the present work, various computational strategies are developed for incremental inelastic analysis of plane frames considering the existence of geometrical nonlinearity, combined with strain hardening or softening effects and non-holonomic behaviour. The modes of deformation are expressed through the natural approach theory, whereas the motion of the material is described by the co-rotational formulation. The elastic tangent stiffness matrix of the element is obtained following an exact solution of the differential equation of a beam-column subjected to end forces. Inelasticity is taken into account by adopting a plastic hinge model. The resulting system of nonlinear equations is solved iteratively inside an incremental loading step using the spherical arc-length technique.

Several computational difficulties, regarding the nonholonomic behavior and the load increment control for inelastic materials with hardening or softening behavior, are addressed. Local unloading is usually materialized, in various computer programs, through a stress-strain relation over the beam cross-section. This has an effect of increasing the computational effort considerably. In this work an alternative approach based on the plastic hinge model, is proposed. Local unloading occurs whenever the product of the increments of the plastic deformations with the existing stress state is negative. As far as the adjustment of the load increment, the radius of the arc, used by the arc-length method, decreases every time more than one plastic hinge forms. Thus only one plastic hinge is allowed to open inside an iteration, within a loading step. Some preliminary results holding for elastic perfectly plastic materials behavior, has already been presented in [2].

Additionally, it was observed that the existing software packages confront severe issues, in

case one combines a yield surface criterion (axial force and moment interaction surface) with a hardening or softening behavior. In order to fulfill the required equilibrium condition for the convergence of the increment, after the yield has occurred, usually the axial force and the moment are decoupled [3]. As a result, the yield surface criterion has been substituted with monotonic laws for the axial force and the moment, which are defined by the user. On the contrary, in the current work, the axial force and the moment are not treated separately. As soon as a plastic hinge at one end of the element has opened, the force vector is projected on the plastic strain vector. In this way both the values of the axial force and the moment are simultaneously corrected and the consistency condition is fulfilled. This return mapping rule proves to be quite stable and it has been modified to cater also for the case where a plastic hinge appears at the other end of the element.

The whole methodology renders a quite accurate and efficient numerical procedure. Application examples are solved, which show that the inclusion of plastic unstressing may totally differentiate the structural behaviour.

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