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ON THE COSSERAT-CAUCHY HOMOGENIZATION PROCEDURE FOR HETEROGENEOUS PERIODIC MEDIA

D. Addessi*[1], M.L. De Bellis[1], E. Sacco[2]

[1] Dip. di Ingegneria Strutturale e Geotecnica,
Universitá di Roma "Sapienza", Via Eudossiana 18, 00184, Roma, Italy
[2] Dip. di Ingegneria Civile e Meccanica,
Universitá di Cassino e del Lazio Meridionale, Via G. Di Biasio 43, 03043, Cassino (FR), Italy
daniela.addessi@uniroma1.it, marialaura.debellis@uniroma1.it,
sacco@unicas.it

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In the last decades the use of different and innovative composite materials has turned out to be a significant challenge, due to their wide use in many fields of engineering, such as structural, mechanical, automotive and aerospace engineering. In this respect, the development of effective computational procedures for analyzing composite materials plays a crucial role for reproducing their structural responses and designing new materials with optimized properties.

A wide variety of approaches was proposed in literature. Among them, in this work the attention is focused on the computational homogenization procedures coupling a Cosserat continuum at the macro level with a Cauchy continuum at the micro-level for modeling periodic 2D composite materials in the linear elastic range, as discussed in [3]. To this end, Addessi et al. [1] recently proposed a higher-order kinematic map linking the micro- and macro-levels in a displacement-based procedure. Moreover, suitable boundary conditions on the Unit Cell (UC) selected for representing the heterogeneous medium at the micro-level are formulated.

The first part of the paper is devoted to the characterization of the perturbation fields that arise in the UC when the higher-order polynomial boundary conditions are imposed. In particular, two techniques are compared.

The first one is based on the derivation of proper boundary conditions to be applied on the single UC in order to reproduce the response of the overall medium when the boundary conditions are defined on the basis of the Cosserat macroscopic strain components [1].

The second one represents a new micromechanical approach, the so-called *three-steps computational homogenization*. The basic idea, firstly introduced by Yuan et al. [4] for the second order asymptotic homogenization and subsequently modified by Bacigalupo and Gambarotta [2], is that the perturbation displacement field can be decomposed into three

different contributions; they depend on the first, second and third order gradient of the kinematic map. It is thus possible to derive explicit relations between the perturbation displacement inside the UC and the macroscopic strain components.
Comparisons of the results obtained considering the two approaches for a composite characterized by cubic symmetry are performed. In particular, it emerges that adopting the three-steps computational homogenization a better estimation of the reference solution is achieved.
The second part of the paper is, instead, focused on the identification of the homogenized linear elastic constitutive parameters. Reference is made to the additional Cosserat components, thus relating the skew symmetric shear strain and the two curvature components to the work-conjugated stresses, for the selected two-phase composite material. A standard procedure, based on the application of the Hill-Mandel macro-homogeneity condition, is adopted. The purpose is to highlight some typical aspects and some limits widely discussed in the literature, [5]. It emerges that the constitutive response of the homogenized UC depends on the particular choice of the cell itself at odds with what expected. In order to obtain an objective response it appears necessary to analyze a set of UCs, then performing the identification of the macroscopic constitutive coefficients on the basis of the average response of the different UCs.

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