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RATE OPTIMALITY OF ADAPTIVE ALGORITHMS PART I: AXIOMS OF ADAPTIVITY

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Adaptive mesh-refining algorithms dominate the numerical simulations in computational sciences and engineering because they promise optimal convergence rates in an overwhelming numerical evidence. The mathematical foundation of optimal convergence rates has recently been completed and shall be discussed in this presentation. We aim at a simultaneous axiomatic presentation of the proof of optimal convergence rates for adaptive finite element methods as well as boundary element methods in the spirit of [1]. For this purpose, an overall set of four axioms on the error estimator is sufficient and (partially even) necessary.

The abstract setting concerns an estimator $\eta(\mathcal{T})$ for each admissible triangulation, written $\mathcal{T} \in \mathbb{T}$. This estimator allows a piecewise estimation in the sense that

$$\eta(\mathcal{T})^2 = \sum_{T \in \mathcal{T}} \eta(\mathcal{T}; T)^2$$

and hence the Dörfler marking in the sense that the selected set $\mathcal{M}$ on the current level ℓ of the successive adaptive mesh-refining satisfies

$$\Theta\, \eta(\mathcal{T})^2 \leq \sum_{T \in \mathcal{M}} \eta(\mathcal{T}; T)^2$$

for some sufficiently small and positive bulk parameter $0 < \Theta < 1$. Given the adaptive algorithm, the computed sequence of triangulations $(\mathcal{T}_\ell)$ and its estimators $(\eta_\ell) := (\eta(\mathcal{T}_\ell))$, are compared with the optimal choices amongst the triangulations

$$\mathbb{T}(N) := \{\mathcal{T} \in \mathbb{T} : |\mathcal{T}| \leq |\mathcal{T}_0| + N\}$$

with at most N of refined element domains; $\mathcal{T}_0$ is the initial triangulation which generates the set $\mathbb{T}$ of admissible triangulations by newest-vertex bisection and $|\bullet|$ denotes the counting measure for finite sets which gives the number of simplices $|\mathcal{T}|$ in the triangulation $\mathcal{T}$.

The presentation discusses four axioms which are formulated for the estimators $(\eta) := (\eta(\mathcal{T})_{\mathcal{T}\in\mathbb{T}}$ only and which guarantee the optimal rates in the sense that, for any rate $0 < \sigma < \infty$,

$$|\eta|_\sigma := \sup_{N\in\mathbb{N}_0} (1+N)^\sigma \min_{\mathcal{T}\in\mathbb{T}(N)} \eta(\mathcal{T}) < \infty$$

implies that the computed values of the estimator converge with the same rate in the sense that

$$\sup_{\ell\in\mathbb{N}_0} (1 + |\mathcal{T}_\ell - |\mathcal{T}_0||)^\sigma \eta_\ell \leq C\,|\eta|_\sigma$$

for some universal constant $C = C(\mathcal{T}_0, \sigma, \Theta)$ and sufficiently small Θ.

The four axioms are estimator stability on non-refined elements (A1), reduction on refined elements (A2), quasi-orthogonality (A3), and discrete reliability (A4). The presentation shall discuss those properties in very simple examples and motivate the different arguments which leads to the aforementioned optimal convergence rates. It can be shown that the convergence rates of the estimators leads to optimality in terms of nonlinear approximation classes in case of efficiency and so discovers the former results from the literature, e.g., [2].

The presentation by Dirk Praetorius shall discuss more examples for adaptive finite and boundary element methods.

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