

## EFFICIENT AND HIGHLY ACCURATE HIGH ORDER MESHLESS METHODS BASED ON HU-WASHIZU VARIATIONAL PRINCIPLE

Qinglin Duan<sup>1</sup>, Xin Gao<sup>2</sup>, Bingbing Wang<sup>3</sup> and Xikui Li<sup>4</sup>

<sup>1</sup> Engineering Mechanics, Dalian University of Technology, China, qinglinduan@dlut.edu.cn

<sup>2</sup> Engineering Mechanics, Dalian University of Technology, China, gaox07@mail.dlut.edu.cn

<sup>3</sup> Engineering Mechanics, Dalian University of Technology, China, bingbkwang@gmail.com

<sup>4</sup> Engineering Mechanics, Dalian University of Technology, China, xikuili@dlut.edu.cn

**Key Words:** *Meshless, Meshfree, EFG, Hu-Washizu.*

Due to the non-polynomial character of meshfree approximations, such as the moving least-squares (MLS) employed in the element-free Galerkin (EFG) method, a large number of evaluation points are required in the evaluation of the Galerkin weak form in meshless methods, especially for high order approximations. This leads to the one of the major drawbacks of meshless Galerkin methods, i.e. the low computational efficiency. In addition, even with so many sampling points, conventional meshless Galerkin methods which are based on the classical one-field variational principle are still not able to exactly pass patch tests.

In this work, we present a novel EFG formulation based on the Hu-Washizu three-field variational principle. Its outstanding merits are that it uses much less evaluation points and meanwhile it can exactly, in a numerical sense, pass the corresponding patch tests. We name it as consistent EFG (CEFG) method. The proposed CEFG method also provides the theoretical foundation for the quadratically consistent 3-point (QC3) [1] and 1-point (QC1) [2] integration schemes we previously developed for meshless Galerkin methods.

Numerical integration schemes with corrected nodal derivatives at quadrature points are proposed for linear, quadratic and cubic CEFG method according to the satisfaction of the orthogonality condition between stress and strain difference in the framework of the Hu-Washizu variational principle. The consistency of the corrected nodal derivatives and the satisfaction of patch test conditions are theoretically proved and also numerically validated. Numerical results show that the proposed CEFG method greatly improves the numerical performance of the EFG method in terms of accuracy, convergence, efficiency and stability, especially the proposed cubic CEFG method which shows amazing accuracy and convergence.

### REFERENCES

- [1] Q. Duan, X. Li, H. Zhang and T. Belytschko, Second order accurate derivatives and integration schemes for meshfree methods. *International Journal for Numerical Methods in Engineering*, Vol. 92, pp. 399–424, 2012.
- [2] Q. Duan, X. Li, H. Zhang, B. Wang and X. Gao, Quadratically consistent one-point (QC1) quadrature for meshfree Galerkin methods. *Computer Methods in Applied Mechanics and Engineering*, Vol. 245, pp. 256–272, 2012.