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A DISCRETE VARIATIONAL APPROACH TO NON-SMOOTH DYNAMICS AND OPTIMAL CONTROL

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The optimal control of human jumping and walking movements requires simulation techniques, which handle the contact's establishing and releasing between the foot and the ground. The characteristics of human jumping are analysed using a simplified monopedal jumper, modelled as a three-dimensional constrained multibody system. The model consists of four rigid bodies, which represent the foot, the calf, the thigh and the upper part of the body, see Fig. 1 (left). The human knee joint is modelled as a revolute joint, while the ankle and the hip are modelled as spherical joints. The system's dynamics changes inherently between the contact phases (jump off and landing) and the flight phase. More precisely, two contact points are considered one at the heel and one at the forefoot. The contact to the ground is modelled as a perfectly plastic impact, i.e. the contact point is fixed on the ground. An inequality constraint ensures that the corresponding contact force prevents the penetration of the ground, not the release of contact, see [2] and Fig. 2.

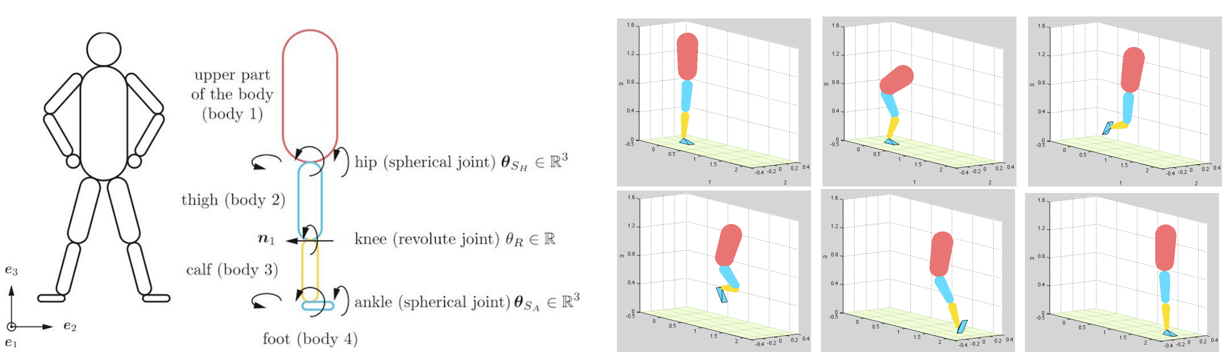

Figure 1: Four link model of the monopedal jumper (left) and snapshots of an optimised longjump motion (right)

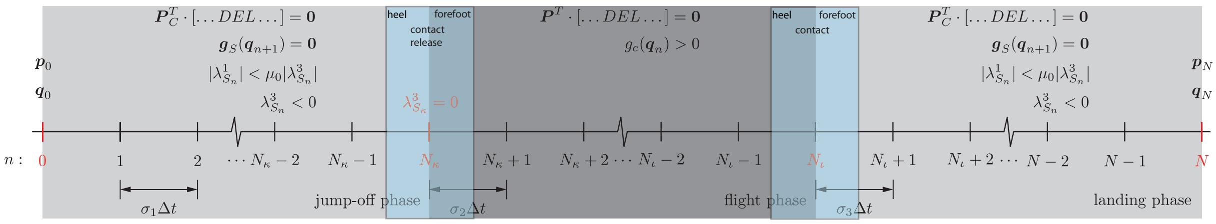

Figure 2: Jump-off, flight and landing phase during a long jump and corresponding constraints on the optimisation

The direct transcription method DMOCC, see [3], is used to transform the optimal control problem into a constrained optimisation problem. It involves a mechanical integrator based on a discrete constrained version of the Lagrange-d'Alembert principle. This integrator represents exactly the behaviour of the analytical solution concerning the consistency of momentum maps and symplecticity. To guarantee the structure preservation and the geometrical correctness during the establishing and releasing of contacts, the non-smooth problem is solved including the computation of the contact or contact release configuration as well as the contact time and force, instead of relying on a smooth approximation of the contact problem via a penalty potential. Snapshots of a longjump motion with minimal control effort are shown in Fig. 1 (right).

The monopedal jumper can be considered as an examples of a hybrid system, where the dynamics is subject to an inherent switch due to the closing or opening of contacts. While in a first approach, the sequence (not the switching time) in which the contact and flight phase follow each other is considered as known, a more general approach is the optimisation of the whole locomotion requiring a combined model including transitions between the different dynamical systems. Integer valued functions can be used to control if and when the switch to another dynamical system occurs, i.e. they permit to control the sequence and switching times of the dynamical systems. Similar to [1], a variable time transformation allows to eliminate the integer valued functions and therefore to apply gradient based optimisation methods to approximate the mixed integer optimal control problem.

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