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# CALCULATIONS OF FREE VIBRATION FREQUENCIES FOR THIN MICROSTRUCTURED PLATE BANDS BY ASYMPTOTIC-TOLERANCE AND TOLERANCE MODELS

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The main problem under consideration in this note is the effect of the microstructure size on free vibrations of thin functionally graded plate bands with a tolerance-periodic microstructure in planes parallel to the plate midplane. These plate bands have the microstructure along their span $L$, i.e. along one direction, $x_1$, but along the perpendicular direction plate properties are constant, cf. Fig. 1. Moreover, the size of the microstructure $l$ is assume to be the same order as the plate thickness $d$, cf. Mazur-Śniady et al. [4]. The general problems of modelling of these plates were shown by Jędrysiak [1], but similar functionally graded plates with the plate thickness being small compared to the microstructure size were analysed by Kaźmierczak and Jędrysiak [2].

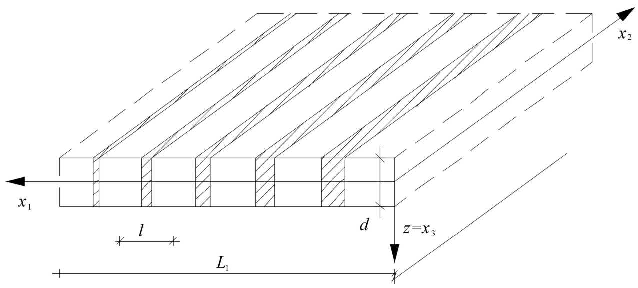


*Fig. 1. A fragment of a microstructured plate band under consideration.*

Free vibrations of these plate bands are described by partial differential equations, with highly oscillating, tolerance-periodic, non-continuous coefficients, which are not a good tool to analyse various special problems of these plates. Plates of this kind are often treated to be made of *a functionally graded material*, cf. Suresh and Mortensen [6], and here are called *microstructured functionally graded plates*. Various averaged models are proposed, in which equations with smooth, slowly-varying coefficients replace the abovementioned equations. The models are based on the known methods, proposed for macroscopically homogeneous composites, cf. the books [6, 8], e.g. on the asymptotic homogenization method for periodic plates, cf. Kohn and Vogelius [3]. Many theoretical and numerical results of various problems of functionally graded structures are presented in many papers. A collocation method with higher-order plate theories was applied to analyse vibrations of functionally graded plates by Roque et al. [5]. A dynamic behaviour of functionally graded shells was investigated by Tornabene and Viola [7]. An extended list of papers related to various thermomechanical problems of functionally graded structures can be found in [6, 8].

Unfortunately, governing equations of these models neglect usually the effect of the microstructure size. However, the microstructure size can be described using models obtained in the framework of *the tolerance modelling method* (cf. Woźniak et al. (eds.) [8]). This method was applied to describe various problems of dynamics and stability for periodic structures and thermomechanical problems for periodic composites in a series of papers, e.g. for periodic thin plates with the microstructure size of an order of the plate thickness by Mazur-Śniady et al. [4]. A list of references can be found in the aforementioned book [8].

The tolerance approach is also applied to analyse thermomechanical problems for functionally graded structures, e.g. for transversally graded thin plates by Kaźmierczak and Jędrysiak [2]. Some other examples of applications of the tolerance method for these composites and structures can be found in the book by Woźniak et al. (eds.) [8].

The first aim of this note is to derive equations of *the tolerance model* and *the asymptotic-tolerance model* for *the microstructured functionally graded plates*, which have slowly-varying coefficients in $x_1$ and take into account the microstructure size. These equations are obtained using the tolerance and the asymptotic modelling, [2]. Formulated in [8, 2] concepts and assumptions are applied, e.g.: a slowly-varying function, a tolerance-periodic function or an averaging operation, defined on an interval $(-l/2,l/2)$. The main modelling assumption, called *the micro-macro decomposition*, is that the plate displacements $u_i(\mathbf{x},z,t)$, $i$=1,2,3, are decomposed in the form: $u_3(\mathbf{x},z,t)=w_3(\mathbf{x},t)$, $u_\alpha(\mathbf{x},z,t)=-[\partial_\alpha w_3(\mathbf{x},t)+h(x_1)v_\alpha(\mathbf{x},t)]z$, where: $h(\cdot)$ is the known *fluctuation shape function* (being a tolerance-periodic in $x_1$); $w_3(\cdot,x_2,t)$ and $v_\alpha(\cdot,x_2,t)$, $\alpha$=1,2, are *macrodeflections* and *fluctuation amplitudes* of the plate midplane, respectively, being slowly-varying functions in $x_1$. The second aim is to analyse free vibration frequencies of microstructured plate bands for different functions describing a distribution of plate properties and various boundary conditions. Because the model equations have functional coefficients to calculate these frequencies some analytical approximate methods can be used, e.g. the Ritz or the orthogonalisation method.

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