

## **CRACK PROPAGATION MODELLING USING THE SCALED BOUNDARY FINITE ELEMENT METHOD: A FINITE FRACTURE MECHANICS APPROACH**

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The two criteria for the direction of crack propagation viz., the maximum circumferential stress criterion and maximum energy release rate criterion are widely used in failure analyses of structures. The maximum circumferential stress criterion is usually used in crack-free structures. On the other hand, the energy criterion is more suitable for structures containing large cracks. However, application of these criteria to analyse failure of structures containing more complicated geometries i.e., notches, holes and material interfaces have not been very successful [1,2]. For such geometries, the asymptotic behaviour in the vicinity of the crack tip is more complex than that of cracks in homogeneous materials. To analyse the failure of structures containing such features, Cornetti et al. [2] proposed a coupled criterion that considers both the stress and energy release at the stress concentration. This criterion is developed based on the finite fracture mechanics [3]. It assumes that a crack of finite length initiate from a stress concentration. The stress and energy release rate averaged over this finite length are used to determine the conditions for failure. Various studies have shown that the finite fracture mechanics can be successfully used to compute the crack initiation parameters of various kinds of stress concentrations i.e., interface cracks [4,5] and sharp V-notches [1].

Finite fracture mechanics is usually implemented in numerical methods. The available literature reports the implementation of finite fracture mechanics within the framework of the finite element method (FEM) [6-9]. A finite element implementation of finite fracture mechanics necessitates very fine meshes in the vicinity of the stress concentrations because the polynomial basis functions in the FEM are inadequate to model the asymptotic behaviour of the stress fields therein. The scaled boundary finite element method (SBFEM) is a semi-analytical approach developed by Song and Wolf [10], which is very efficient in solving problems with singularities. This paper discusses the application of finite fracture mechanics within the framework of the SBFEM for crack propagation modelling.

Application of the SBFEM via finite fracture mechanics has two significant advantages. First, asymptotic fields of any kind near the stress concentrations are analytically represented by the SBFEM [11]. The energy release rate can be conveniently expressed as an analytical integral in the form of matrix power functions. Second, a finite crack extension in any direction can be modelled conveniently in the SBFEM. Specifically, the SBFEM can be formulated on

polygons of arbitrary number of sides. A crack can be modelled by a single polygon [10]. Introducing a finite crack extension only necessitates splitting the cracked polygon into two. When a crack propagates, the simple, yet flexible remeshing algorithm developed by Ooi et al. [12] can be used to propagate the crack. This algorithm makes only minimal changes to the global mesh.

The efficiency of the developed method is demonstrated by modelling crack propagation in few V-notched specimens.

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