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ON A CONSISTENT APPLICATION OF NEWTON'S LAW TO MECHANICAL SYSTEMS WITH MOTION CONSTRAINTS

Sotirios Natsiavas* and Elias Paraskevopoulos
Dept. of Mechanical Engineering, Aristotle University, Thessaloniki, Greece, natsiava@auth.gr

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In engineering applications, the configuration of a system is described by a set of generalized coordinates. Frequently, a minimum number of them is chosen [6]. However, in many cases it is beneficial to use more coordinates than those actually needed. Then, extra equations are also introduced, representing the effect of motion constraints [12]. These equations complement the set of second order ordinary differential equations (ODEs), arising by application of the law of motion to all components of the system. This gives rise to a set of differential-algebraic equations (DAEs), leading to severe difficulties during their numerical solution [2]. A comprehensive review on these problems can be found in [1, 5].

The main objective of this work is to present some representative results obtained by numerical integration of a new set of equations of motion for mechanical systems with scleronomic constraints. The new approach is based on some fundamental concepts of differential geometry and treats both holonomic and nonholonomic constraints by considering them as part of the overall process of deriving the equations of motion [10]. This leads to an assignment of inertia, damping and stiffness properties to the constraints. As a result, the equations of motion are second order ODEs in both the generalized coordinates and the Lagrange multipliers. Consequently, there is no need to introduce artificial parameters for scaling and stabilization, as required by DAE or penalty based formulations. In addition, the geometric properties of the original manifold are kept unchanged by the additional constraints. This preserves the geometric properties of special curves of the manifold employed in the numerical discretization and leads to major advantages compared to previous work in the field of computational Multibody Dynamics [3, 4, 9].

Some typical results are presented next. Specifically, in Figure 1 are compared results obtained by direct integration of the new set of equations of motion with those obtained by a state of the art code [7]. In brief, this code sets up a system of DAEs and solves them numerically. The results of the new method are labeled by LMD, while those obtained by applying classical index-3, stabilized index-2 or index-1 methods of the code are labeled by I3, SI2 and SI1, respectively. Moreover, the numbers in the labels correspond to an allowable error in the calculations. Quite similar results with those of [7] were also obtained by [8].

The results refer to torque free motion of a rigid spinning rod with unit mass and principal mass moments of inertia equal to 50, 50 and 1 kgm^2 with respect to the axes of a body frame attached to the center of mass of the rod. The rod is supported by a spherical joint at a point with position (-0.1, 0, 0) m relative to its center of mass. In addition, it is set in motion by an initial velocity (0, 5, 0) m/s to its center of mass, together with an angular velocity (1, 0, 50) rad/s to the body. In Fig. 1a is depicted the kinetic energy of the rod. The kinetic energy obtained by the DAE methods decreases with time, while the new approach captures the correct constant value for this quantity. Also, in Fig. 1b is shown history of the magnitude of the angular momentum of the rod. Again, the classical numerical methods predict a monotonically decreasing value with time, in contrast to the predictions of the new approach.

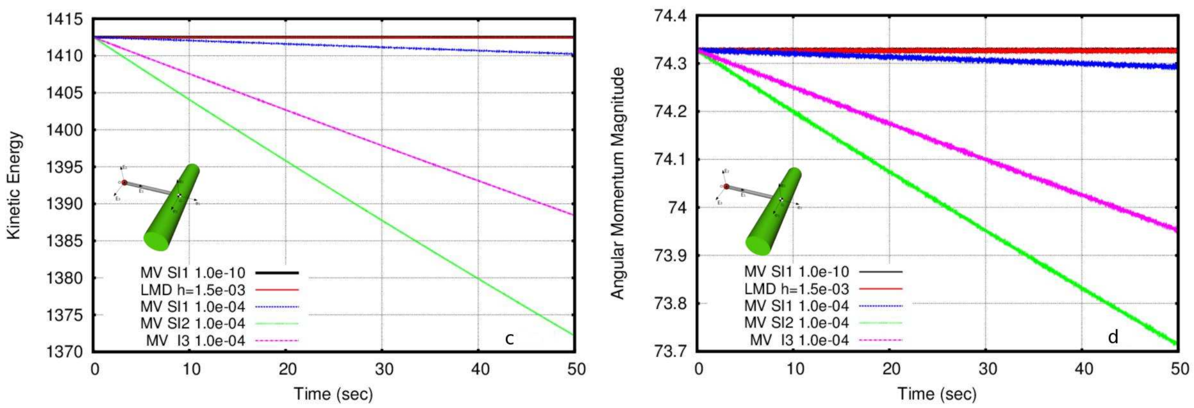

Figure 1: Typical numerical results for a torque free motion of a rod

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