

## **An Embedded Crack in a Functionally Graded Orthotropic Coating Bonded to a Homogeneous Substrate under a Frictional Hertzian Contact**

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We consider the elasto-static problem of an embedded crack in a graded orthotropic coating bonded to a homogeneous substrate subject to statically applied normal and tangential surface loading. The crack direction is parallel to the free surface. The coating is graded in the thickness direction and is orthogonal to the crack direction. This coating is modelled as a non-homogeneous medium with an orthotropic stress-strain law. The equivalent crack surface stresses are first obtained and substituted in the plane elasticity equations. Using integral transforms, the governing equations are converted into singular integral equations which are solved numerically to yield the displacement field as well as the crack-tip Stress Intensity Factors. We present a theoretical formulation for the problem in the static case. A numerical predictive capability for solving the singular integral equations and computing the crack-tip stress intensity factors is proposed. Since the loading is compressive, a previously developed crack-closure algorithm [1] is applied to avoid interpenetration of the crack faces. The main objective of the work is to investigate the effects of the material orthotropy and non-homogeneity of the graded coating on the crack-tip stress intensity factors, with and without using the crack-closure algorithm, for the purpose of gaining better understanding on the behavior and design of graded coatings.

### **REFERENCES**

- [1] El-Borgi, S., Erdogan, F. and Hidri, L., "A partially insulated embedded crack in an infinite functionally graded medium under thermo-mechanical loading". *International Journal of Engineering Science* 42, 371-393, 2004.