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ADAPTIVE TIME STEPPING AND SCHWARZ WAVEFORM RELAXATION METHOD FOR COMPRESSIBLE NAVIER–STOKES EQUATIONS

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This article focuses on the research field of laminar flow of an ideal gas. We propose an improved parallel time-space method for steady/unsteady problems modelised by Euler and Navier-Stokes equations for a direct numerical simulation. A space-time domain decomposition algorithm for the compressible Navier–Stokes problem has been designed, with the aim of implementing it in three dimensions, in an industrial code. The system is discretised with a second order implicit scheme in time and Finite Volumes Method in space. To achieve full speedup performance, a Schwarz Waveform Relaxation (SWR) method is used, as it allows local space and time stepping. We compare a second order in time Schwarz- Backward Differentiation Formula (S-BDF2) method with fixed step size method and a new, adaptive time stepping Schwarz Waveform Relaxation with the same implicit time scheme method (SWR-BDF2). We illustrate the advantages of SWR-BDF2 and the considerable cost savings of adaptive time stepping with a diminishing number of coupling iterations. The performances of different parallel computing strategies (using OpenMP and MPI) are compared for complex configurations.

Domain decomposition methods split large problems into smaller sub-problems that can be solved in parallel. Usually, only space domain decomposition method is used to provide high-performing algorithms in many fields of numerical applications. To achieve full performance on large clusters with up to 100 000 nodes (such as recently the IBM Sequoia, or GPUs) requires another dimension to parallelize. An essential gain to be obtained from time-space domain decomposition is the ability to apply different time-space discretisation on sub-domains thus improving efficiency and convergence of implicit schemes.
The Schwarz Waveform Relaxation (SWR) domain decomposition methods based on [1] and developed in [2] is one method to realize this goal. Initially analyzed for linear

problems, it has been extended to nonlinear cases. We choose to use a second order Backward Differentiation Formula with a Schwarz decomposition that divides the system in several local overlapping subsystems. Each subsystem is solved in parallel and then the whole solution is rebuilt through proper continuity boundary conditions.
Within the SWR iterative process, we propose an adaptive time stepping technique to improve the scheme consistency. By adaptivity we mean that, for each SWR iteration, as we improve the coupling conditions we adapt the time step to satisfy the CFL condition and ensure stability of our method. Space discretisation is achieved with finite volumes on cartesian non-conforming grids. The sub-domains overlap region has the stencil size. We use a second order projection method to exchange data in time and in space.
Performances of the different parallel computing strategies (using OpenMP and MPI) are first compared on the viscous motion of a 2D isolated vortex in an uniform free-stream based on H.C. Yee [3] and secondly on the mixing layer case. We investigate the effects of adaptive time-stepping on the computational stability of our codes. The solutions compare favorably in terms of accuracy, and we observe a substantial saving in number of Schwarz iterations taken with adaptive time stepping over fixed step size.

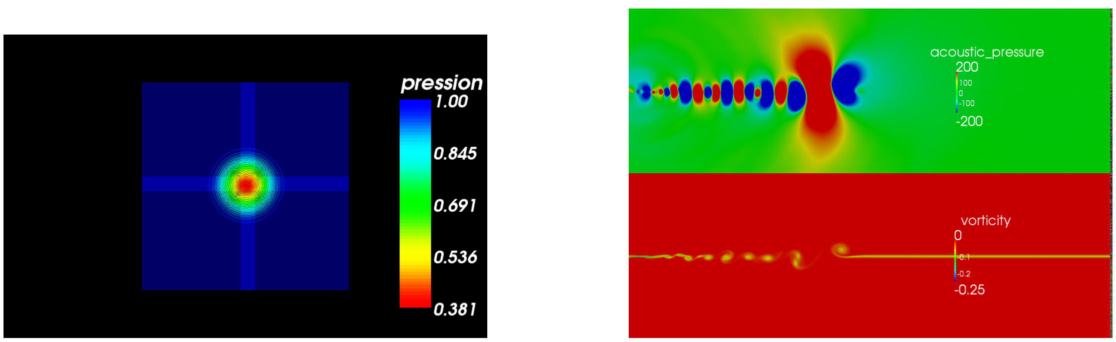

Figure 1: (left) 2D Isolated Vortex, (right) 2D Mixing Layer

REFERENCES

[1] E. Lelarasmee, A. E. Ruehli, and A. L. Sangiovanni-Vincentelli. *The Waveform Relaxation method for time-domain analysis of large scale integrated circuits.* IEEE Trans. CAD IC Systems, 1982.

[2] J. Gander and L. Halpern. *Optimized Schwarz Waveform Relaxation methods for advection reaction diffusion problem.* SIAM Journal on Numerical Analysis, Vol. **45**, no 2, 2008.

[3] H.C.Yee, N.D. Sandham and J. Djomehri.*Low dissipative high order shock-capturing methods using characteristic-based filters.* J. Comp. Phys., Vol. **150**, 199-238, 1999.