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ON THE BLENDING OF REGULARIZATION AND LARGE-EDDY SIMULATION MODELS

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The incompressible Navier-Stokes (NS) equations form an excellent mathematical model for turbulent flows. In primitive variables they read

$$\partial_t \boldsymbol{u} + \mathcal{C}(\boldsymbol{u},\boldsymbol{u}) = \mathcal{D}\boldsymbol{u} - \nabla p\,; \quad \nabla\cdot\boldsymbol{u} = 0, \tag{1}$$

where $\boldsymbol{u}$ denotes the velocity field, p represents the pressure, the non-linear convective term is defined by $\mathcal{C}(\boldsymbol{u},\boldsymbol{v}) = (\boldsymbol{u}\cdot\nabla)\,\boldsymbol{v}$, and the diffusive term reads $\mathcal{D}\boldsymbol{u} = \nu\Delta\boldsymbol{u}$, where ν is the kinematic viscosity. Since direct numerical simulations of turbulent flows cannot be computed at high Reynolds numbers, a dynamically less complex mathematical formulation is needed. The most popular example thereof is the Large-Eddy Simulation (LES). Alternatively, regularizations of the non-linear convective term basically reduce the transport towards the small scales: the convective term in the NS equations, $\mathcal{C}$, is replaced by a smoother approximation [1, 2, 3]. In our previous works (see [4] and references therein), we restricted ourselves to the $\mathcal{C}_4$ approximation [3]: the convective term in the NS equations (1) is replaced by the following $\mathcal{O}(\epsilon^4)$-accurate smoother approximation

$$\mathcal{C}_4(\boldsymbol{u},\boldsymbol{v}) = \mathcal{C}(\overline{\boldsymbol{u}},\overline{\boldsymbol{v}}) + \overline{\mathcal{C}(\overline{\boldsymbol{u}},\boldsymbol{v}')} + \overline{\mathcal{C}(\boldsymbol{u}',\overline{\boldsymbol{v}})}, \tag{2}$$

where the prime indicates the residual of the filter, *e.g.* $\boldsymbol{u}' = \boldsymbol{u} - \overline{\boldsymbol{u}}$, which can be explicitly evaluated, and $\overline{(\cdot)}$ represents a symmetric linear filter with filter length ϵ. However, two main drawbacks were observed: (i) due to the energy conservation, the model solution tends to display an additional hump in the tail of the spectrum (see Figure 1) and (ii) for very coarse meshes the damping factor can eventually take very small values.

In this context, we propose to combine regularization and LES modeling. The linkage follows from (approximately) restoring the Galilean invariance for the regularization by means of a modification of the diffusive term. Shortly, by imposing all the symmetries and conservation properties of the original convective operator, $\mathcal{C}(\boldsymbol{u},\boldsymbol{u})$, and canceling the second-order terms leads to the following one-parameter fourth-order regularization

$$\partial_t \boldsymbol{u}_\epsilon + \mathcal{C}_4^\gamma(\boldsymbol{u}_\epsilon,\boldsymbol{u}_\epsilon) = \mathcal{D}_4^\gamma\boldsymbol{u}_\epsilon - \nabla p_\epsilon\,; \quad \nabla\cdot\boldsymbol{u}_\epsilon = 0, \tag{3}$$

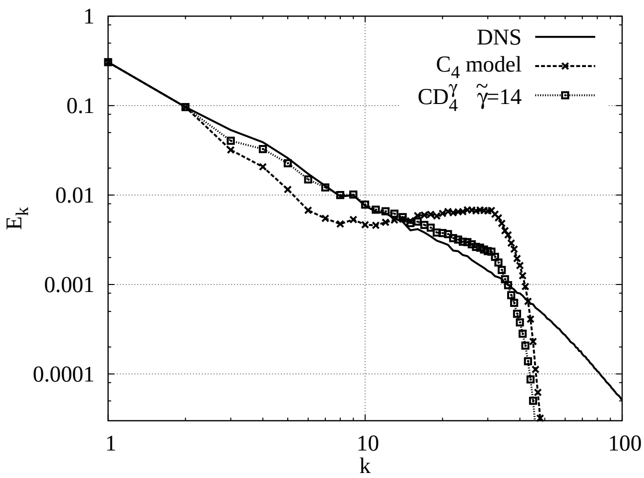

Figure 1: Energy spectra for a forced homogeneous isotropic turbulence at $Re_\lambda = 202$. Comparison of the DNS results with the models $\mathcal{C}_4$ and $\mathcal{CD}_4^\gamma$ (with $\tilde{\gamma} = 14$) using a 64^3 mesh.

where the convective and the diffusive terms are modified in the same vein

$$\mathcal{C}_4^\gamma(\boldsymbol{u},\boldsymbol{v}) = \frac{1}{2}((\mathcal{C}_4 + \mathcal{C}_6) + \gamma(\mathcal{C}_4 - \mathcal{C}_6))(\boldsymbol{u},\boldsymbol{v}) \quad \text{and} \quad \mathcal{D}_4^\gamma \boldsymbol{u} = \mathcal{D}\boldsymbol{u} + \tilde{\gamma}(\mathcal{D}\boldsymbol{u}')', \tag{4}$$

where $\tilde{\gamma} = 1/2(1+\gamma)$ and $\mathcal{C}_6(\boldsymbol{u},\boldsymbol{v}) = \mathcal{C}(\overline{\boldsymbol{u}},\overline{\boldsymbol{v}}) + \mathcal{C}(\overline{\boldsymbol{u}},\boldsymbol{v}') + \mathcal{C}(\boldsymbol{u}',\overline{\boldsymbol{v}}) + \overline{\mathcal{C}(\boldsymbol{u}',\boldsymbol{v}')}$. Notice that in this case the dissipation is reinforced by means of an hyperviscosity term. As expected, this basically acts at the tail of the energy spectrum; therefore, it helps to mitigate the two above-mentioned drawbacks. From a LES point-of-view, we can relate the $\mathcal{CD}_4^\gamma$ regularization to a closure models for any invertible filter. Then, to apply the method two parameters still need to be determined; namely, the local filter length, ϵ, and the constant $\tilde{\gamma}$. The former follows from the criterion that the vortex-stretching mechanism must stop at the smallest grid scale [4]. The latter can be approximately bounded by assuming that the smallest grid scale lies within the inertial range for a classical Kolmogorov energy spectrum [5]. Doing so, the following bound follows $\tilde{\gamma} \gtrsim 4\left(4\sqrt{2}C_K^{-3/2} - 1\right)$, where C_K is the Kolmogorov constant. Simulations for homogeneous isotropic turbulence seems to confirm the adequacy of this bound (see Figure 1). Apart from this, numerical results evaluating the performance of the $\mathcal{CD}_4^\gamma$ method for wall-bounded configurations will be presented during the conference.

REFERENCES

[1] Geurts, B. J. and Holm, D. D. *Physics of Fluids* **15**, L13–L16 (2003).
[2] Guermond, J. L., Oden, J. T., and Prudhomme, S. *Physica D* **177**, 23–30 (2003).
[3] Verstappen, R. *Computers & Fluids* **37**, 887–897 (2008).
[4] Trias, F. X., Gorobets, A., Pérez-Segarra, C. D., and Oliva, A. *International Journal of Heat and Mass Transfer* **57**, 171–182 (2013).
[5] Trias, F. X., Folch, D., Gorobets, A., and Oliva, A. In *Conference on Modelling Fluid Flow 2012* (, Budapest, Hungary, 2012).