

# TOWARDS CONVERGED ADJOINT STATE FOR LARGE INDUSTRIAL CASES BY IMPROVING THE DISCRETIZATION SCHEMES

Mattia Oriani<sup>1\*</sup>, Guillaume Pierrot<sup>2</sup>

<sup>1</sup> Queen Mary, University of London  
327 Mile End Road, E1 4NS London, UK  
mattia.oriani@esi-group.com

<sup>2</sup> ESI Group  
Parc d’Affaires SILIC, 99 Rue des Solets, BP 80112, 94513 Rungis cedex, France  
guillaume.pierrot@esi-group.com

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Despite taking several different approaches, researchers in the field of adjoint computation are currently facing a number of challenges. Problems arise in particular when adjoint methods are used within CFD optimization processes, which involve a nonlinear primal system (i.e. the Navier-Stokes equations).

Examples of such difficulties are: continuous adjoint solvers suffering from convergence issues, suggesting, among other causes, that the discretization schemes and algorithms employed in standard CFD are not always suited to the nature of adjoint equations; discrete adjoint solvers also failing to converge in large cases unless a very accurate solution to the primal problem is provided; adjoint codes produced via reverse Automatic Differentiation being based on hypotheses often not valid [1], thus producing unreliable gradient values as well as being costly in terms of storage memory.

Regardless of the approach, evidence suggests that convergence issues in adjoint solvers are related to the accuracy of the CFD itself, meaning that classical Finite Volume schemes yield a solution that, although acceptable as a mere aerodynamics case study, cannot be used to produce a robust adjoint system. It appears therefore that adjoint solvers could benefit from a well-converged primal flow solution, which in turns requires a) an adequate solving algorithm and b) an appropriate, mathematically sound discretization scheme, which is the aspect we aim to tackle in the present work.

In the past decade, an innovative discretization scheme known as Mixed Finite Volumes

(or Mimetic Finite Differences) has emerged, based on the idea of constructing discrete operators such that they satisfy certain key properties that are satisfied by their continuous counterparts. The scheme allows for more freedom in geometrical properties of mesh elements, exhibits improved accuracy and convergence compared to classical Finite Volumes and has been successfully applied to anisotropic diffusion problems [2], transport equations [3] and Navier-Stokes [4].

We present here a specific implementation of MFV applied to the incompressible Navier-Stokes equations, and we focus our attention on the role it plays within the context of adjoint optimization.

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