

## PARALLEL SOLUTION OF ELASTOPLASTIC PROBLEMS WITH NUMERICAL EXPERIMENTS

M. Čermák<sup>1</sup>, T. Karásek<sup>2</sup>, and M. Merta<sup>3</sup>

<sup>1</sup> IT4Innovations, 17. listopadu 15/2172, 708 33 Ostrava Poruba, Czech Republic,  
martin.cermak@vsb.cz

<sup>2</sup> IT4Innovations, 17. listopadu 15/2172, 708 33 Ostrava Poruba, Czech Republic,  
tomas.karasek@vsb.cz

<sup>3</sup> IT4Innovations, 17. listopadu 15/2172, 708 33 Ostrava Poruba, Czech Republic,  
michal.merta@vsb.cz

**Key words:** *TFETI, Elastoplasticity, Scalability, Parallel Implementation, Numerical Experiments.*

For solving elasticity problems consisting of millions or even hundreds of millions of unknowns domain decomposition methods are very popular among researchers. One of those methods is FETI (Finite Element Tearing and Interconnecting) originally introduced by Farhat and Roux [3]. In their approach a domain is partitioned into non-overlapping subdomains, and an elliptic problem with Neumann boundary conditions is defined for each subdomain. Intersubdomain field continuity is enforced via Lagrange multipliers. The Lagrange multipliers are evaluated by solving a relatively well conditioned dual problem of a small size that may be efficiently solved by a suitable variant of the conjugate gradient algorithm with natural coarse space and lumped preconditioners. In this paper, we use the Total-FETI (TFETI) [2] variant of FETI domain decomposition method, where also the Dirichlet boundary conditions are enforced by Lagrange multipliers. This gives us singular stiffness matrices for all subdomains with a-priori known kernels which are a great advantage in the numerical solution. With known kernel basis the stiffness matrix can be regularize effectively. There are many papers dealing with elasticity problems being solved by FETI or TFETI methods. We extended TFETI method based solver for solving elastoplastic problems and numerical experiments of elastoplastic problems with isotropic and kinematic hardening, and perfect plasticity are presented here. Material models with an associated elastoplasticity with the von Mises plastic criterion and the linear isotropic or kinematic hardening law, or perfect plasticity (see e.g. [1, 4]) are considered. The corresponding elastoplastic constitutive model is discretized by the implicit Euler method in time and consequently a nonlinear stress-strain relation is implemented by the return mapping concept (see e.g. [4, 1]). This approach together with the balance equation, leads to the solution of a nonlinear variational equation with respect to the primal unknown displacement in each time step. The introduced TFETI based algorithm

for the numerical solution of elastoplastic problems was implemented in FLLOP (FETI Light Layer On top of PETSc) which is developed at IT4Innovations national supercomputer center in Ostrava, Czech Republic. Numerical experiments presented in this paper compare different hardening models on 3D elastoplastic benchmarks. A local quadratic convergence of the semismooth Newton method could be observed which corresponds with theoretical results.

## REFERENCES

- [1] Blaheta, R., *Numerical methods in elasto-plasticity*, Documenta Geonica 1998, PERES Publishers, Prague, 1999.
- [2] Dostál, Z., Horák, D., Kučera, R., *Total FETI - an easier implementable variant of the FETI method for numerical solution of elliptic PDE*, Communications in Numerical Methods in Engineering 22 (12), 1155–1162, 2006.
- [3] Farhat, C., Roux, F-X., *An unconventional domain decomposition method for an efficient parallel solution of large-scale finite element systems*, SIAM J. Sci. Stat. Comput. 13, 379–396, 1992.
- [4] de Souza Neto, E. A., Perić, D., Owen, D. R. J., *Computational methods for plasticity: theory and application*. Wiley, 2008.