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Numerical Perturbation Schemes for Convective-Diffusion Equation and Their Applications in NS Equations

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Based on the numerical perturbation reconstruction of convective flux and source term of convective-diffusion equation, the high-order numerical perturbation algorithm is constructed by eliminating truncated error terms in the modified differential equation of perturbation discretized equation. In other words, the high-order spatial step expansion solution of the original discretized equation is obtained by using the numerical perturbation algorithm.

The numerical perturbation upwind and central schemes, including finite difference(FD) and finite volume(FV) schemes, have several advantages: (1) they use less nodes, and they have the same concise form as the first-order upwind scheme and second-order central scheme, respectively, and they are high order accurate and absolutely stable. (2) They have the uniform high order accuracy at boundary and in internal regions. (3) They can effectively reduce the false-diffusion effect caused by the function source term in the convective-diffusion equation, and hence improve the calculation accuracy of convective-diffusion equation with function source terms. While the conventional high-order upwind scheme and central scheme using more nodes are conditional stable, their stable range decreases and easily oscillates with increasing accuracy order. Compared with the numerical perturbation schemes, the conventional high order schemes have to decrease their accuracy order at boundary. The false-diffusion effect caused by source term also affects the numerical accuracy less than their theoretical accuracy. The calculation of 1D and 2D model equations has shown the above mentioned properties.

The momentum equations, energy equation and Species conservation equations in Navier-Stokes (NS) equations are convective-diffusion equation with source terms. The numerical perturbation upwind and central schemes are constructed for NS equations. These schemes coupled with SIMPLE algorithm are used to calculate the incompressible NS equation. Numerical results of the incompressible flows in cavities driven by the lid and buoyant force, flow over a forward and a backward step agree well with their benchmark solution, even provide some new benchmark solutions. The perturbation Roe scheme and perturbation

AUSM scheme are given by the numerical perturbation reconstructions of Roe's scheme and AUSM scheme for compressible flows. These new schemes are used to calculate the 1D compressible flows, the transonic flows over DLR-F6 wing-body and DLR-F6 wing-body-nacelle-pylon, numerical results are more accurate than the original schemes. The perturbation scheme for incompressible flow coupled with the level-set method are also used to simulate two-phase flows. The numerical results of dam break flow, Rayleigh-Taylor instability flow, the lid driven two-phase flow in a cavity and two-phase immiscible flow in microchannels agree well with experiments and the results of others.

REFERENCES

[1] C.A.J. Fletcher, *Computational Techniques for Fluid Dynamics*, Springer, 1997.
[2] Z. Gao, Y.Q. Shen, Analysis and Application of High Resolution Numerical Perturbation Algorithm for Convective-Diffusion Equation, *Chin. Phys. Lett.*, Vol.29, pp. 104702-1-5, 2012.
[3] Z. Gao, Numerical perturbation algorithm and its CFD schemes, *Advances in Mechanics*(in Chinese), Vol. 40, pp. 607-633, 2010.
[4] Z. Gao, G.W. Yang, Perturbation finite volume (PFV) method, *Acta Mechanica Sinica*, Vol. 20, pp.580-590, 2004.
[5] Z. Gao, Y.Q. Shen, G.W. Yang, Perturbation finite volume method for the solution of 2-D Navier-Stokes equations on unstructured collocated meshes, *CFD Jour.*, Vol. 13, pp. 2004-2008, 2004.
[6] Z. Gao, Advances in perturbation finite difference (PFD) method, *Advances in Mechanics*(in Chinese), Vol. 30, pp. 200-215, 2000.
[7] F. J Zeng, S. Shu, M. J. Li, Gao's perturbation finite difference schemes reducing false-diffusion caused by function source term in convective-diffusion type equation, to be published in *J. Hydrodynamics* (series B), 2013.
[8] M.G Dai, Perturbation finite volume scheme and numerical simulation of complex flow based on hybrid grid, Ph.D. thesis(in Chinese), Graduate University of Chinese Academy of Sciences, 2008.
[9] H.F. Dong, The perturbation finite volume algorithm and its applications in two-phase flow computations, Ph.D. thesis (in Chinese), Graduate University of Chinese Academy of Sciences, 2009.