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COMPARISON RESULTS OF FINITE ELEMENT METHODS

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Recent comparison results [1, 2] show that first-order finite element methods (FEMs) for the Poisson model problem are comparable in the sense that their errors on the same mesh are equivalent up to multiplicative constants and higher-order data oscillations. The comparison includes methods that are conceptually very different, e.g., conforming and non-conforming finite elements, mixed methods, discontinuous finite elements, finite volumes and least squares. One might expect that equal-order approximations lead to the same asymptotic convergence rates under uniform mesh refinement. It is, however, very surprising that all those different methods are equivalent even in the pre-asymptotic regime and without any assumptions on the regularity of the solution. The equivalence is preserved under adaptive mesh refinement.

In this talk, we summarize the comparison results and the underlying medius analysis. Furthermore, we discuss the possible equivalence of popular finite element schemes for the Stokes problem including continuous and discontinuous approximations of the pressure variable. Numerical benchmarks illustrate the comparison theorems as well as some conjectures and counter-examples.

This talk is based on joint work with C. Carstensen, K. Köhler and M. Schedensack.

REFERENCES

[1] D. Braess. An a posteriori error estimate and a comparison theorem for the nonconforming P_1 element. *Calcolo*, Vol. **46**(2), 149–155, 2009.

[2] C. Carstensen, D. Peterseim and M. Schedensack. Comparison Results of Finite Element Methods for the Poisson Model Problem. *SIAM J. Numer. Anal.*, Vol. **50**(6), 2803-2823, 2013.

[3] C. Carstensen, K. Köhler, M. Schedensack and D. Peterseim. Comparison Results for the Stokes equations. *Submitted for publication*, 2013.