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A NEW DERIVATION OF PRESSURE POISSON EQUATION IN MOVING PARTICLE SEMI-IMPLICIT METHOD

Motofumi HATTORI[1], Youhei SETA[2], Kazuya SHIBATA[3] and Seiichi KOSHIZUKA[4]

[1] Kanagawa Institute of Technology, Shimo-ogino Atsugi-city 243-0292,
hattori@ic.kanagawa-it.ac.jp
[2] Kanagawa Institute of Technology, 1030 Shimo-ogino Atsugi-city 243-0292,
youhei428428428@hotmail.co.jp
[3] The University of Tokyo, 7-3-1Hongo Bunkyo-ward Tokyo 113-8656,
shibata.kazuya@sys.t.u-tokyo.ac.jp
[4] The University of Tokyo, 7-3-1Hongo Bunkyo-ward Tokyo 113-8656,
koshizuka@sys.t.u-tokyo.ac.jp

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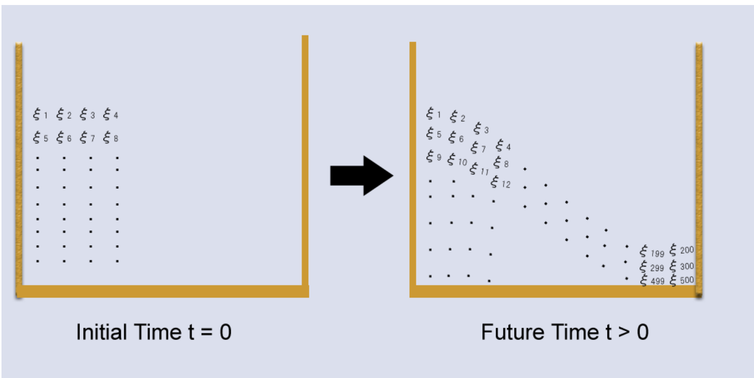

Figure 1: Spatial discretization by many particles.

Let Δt be a sampling time. The position u, the velocity v, and the pressure p are computed at time $t = n\ \Delta t$. $n = 0, 1, 2, \cdots$ become digital times. Let $U[n](\xi)$ be an approximate value for $u(n\ \Delta t,\ \xi)$, let $V[n](\xi)$ be an approximate value for $v(n\ \Delta t,\ \xi)$, let $P[n](\xi)$ be an approximate value for $p(n\ \Delta t,\ \xi)$, and let $\mathrm{Rho}[n](\xi)$ be an approximate value for $\rho(n\ \Delta t,\ \xi)$.

The position $U[n]$, the velocity $V[n]$, and the pressure $P[n]$ should satisfy the discrete time Navier-Stokes equation

$$\frac{V[n+1]-V[n]}{\Delta t} = \frac{\mu}{\text{Rho}[n]}\sum_{i=x,y,z}\frac{\partial^2 V[n]}{\partial U_i[n]^2} - \frac{1}{\text{Rho}[n+1]}\frac{\partial P[n+1]}{\partial U[n+1]} + g \tag{1}$$

$$\frac{U[n+1]-U[n]}{\Delta t} = V[n+1] \tag{2}$$

For these goal equations (1) and (2), the variables $U[n+1]$, $V[n+1]$, $P[n+1]$, and $\text{Rho}[n+1]$ at next time $n+1$ are computed from the variables $U[n]$, $V[n]$, $P[n]$, and $\text{Rho}[n]$ at present time n , as follows.

The temporal velocity V^* and the temporal position U^* are computed

$$\frac{V^*-V[n]}{\Delta t} = \frac{\mu}{\text{Rho}[n]}\sum_{j=x,y,z}\frac{\partial^2 V[n]}{\partial U_j[n]^2} + g \qquad \frac{U^*-U[n]}{\Delta t} = V^* \tag{3}$$

Comparing the discrete time Navier-Stokes equation (1), in order to recover the effect of pressure $P[n+1]$ (unknown) to the left equation of the equations (3), we consider the modifiers V' , U' and the mass density $\text{Rho}^*(\xi)$ as

$$V[n+1] = V^* + V' \qquad U[n+1] = U^* + U' \tag{4}$$

where

$$\frac{V'}{\Delta t} = \frac{-1}{\text{Rho}[n+1]}\ \frac{\partial P[n+1]}{\partial U[n+1]} \qquad \frac{U'}{\Delta t} = V' \qquad \text{Rho}^*(\xi) = \frac{\rho_0}{\det\left(\frac{\partial U^*(\xi)}{\partial \xi}\right)} \tag{5}$$

By adding the equation (3) and the equation (5), we obtain the discrete time Navier-Stokes equation (1) and (2).

Considering the temporal compressibility between between sampling times, we obtain the following pressure Poisson equation

$$\sum_{i=x,y,z}\frac{\partial^2 P[n+1]}{\partial U_i[n+1]^2} = \frac{\rho_0-\text{Rho}^*}{(\Delta t)^2} + \left(\frac{\rho_0-\text{Rho}^*}{\Delta t}\right)\sum_{j=x,y,z}\frac{\partial V_j^*}{\partial U_j^*} \tag{6}$$

which is similar to the one proposed in [2].

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