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GENERALIZED FINITE ELEMENT METHOD (GFEM): ACCURATE AND EFFICIENT COMPUTATION OF THE SOLUTION OF INTERFACE PROBLEMS

Uday Banerjee[1], Keenan Kergrene[2] and Ivo Babuška[3]

[1] Department of Mathematics, Syracuse University, Syracuse, NY 13244, USA,
banerjee@syr.edu and http://math.syr.edu

[2] Département de Génie Mécanique École Normale Supérieure de Cachan, 61, avenue du Président Wilson 94235 Cachan cedex FRANCE, kenan.kergrene@ens-cachan.fr

[3] ICES, University of Texas, Austin, TX 78712-1229, babuska@ices.utexas.edu

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Over the last decade, the GFEM (also known as the eXtended finite element method (XFEM)) has been successfully used to approximate non-smooth solutions of various engineering problems, e.g., the crack propagation problem, interface problems, problems involving voids and inclusions, etc. The GFEM is an extension of the classical finite element method (FEM), where the approximation of the solution is obtained by augmenting the standard finite element space $\mathcal{S}_{FE}$ (piecewise polynomials) with the *enrichment space* $\mathcal{S}_{ENR}$. The accuracy of the GFEM depends on a smart choice of $\mathcal{S}_{ENR}$, incorporating the available information about the unknown solution. However, the associated linear system of the GFEM could be very large, the stiffness matrix could be badly conditioned, and the use of a direct method to solve the linear system could be difficult due to (a) the size of the system, as well as (b) the accumulation of the round-off errors.

In this talk, we will consider the use of GFEM on an interface problem, where the interface is a smooth and closed curve. First part of the presentation will be theoretical, where we will establish the optimal convergence of the GFEM under various variational crimes associated with the replacement of the smooth interface by a closed polygon and the use of "perturbed" enrichments. We will also show that the conditioning of the GFEM depends on the "angle" between the spaces $\mathcal{S}_{FE}$ and $\mathcal{S}_{ENR}$, which in turn depends on the choice of $\mathcal{S}_{ENR}$. For example, GFEMs associated with different enrichment spaces may yield the same approximation property, say $O(h)$, but the GFEM, with the angle between $\mathcal{S}_{FE}$ and its enrichment space $\mathcal{S}_{ENR}$ closest to $\pi/2$, will have the best conditioning and will yield a linear system that is easiest to solve.

In the rest of the talk, we will present various numerical results, where we will compare various GFEMs, associated with various choices of the enrichment space $\mathcal{S}_{ENR}$, with respect to their (a) accuracy, (b) conditioning, and (c) computational effort associated with solving the linear system. These results will indicate the importance of the theory in the understanding of various issues related to GFEM.

As mentioned before, the use of a direct method to solve the linear system of GFEM, which could be extremely large especially in 3D problems, could be prohibitive. We will present an *iterative method*, employing the Schur iteration involving $\mathcal{S}_{FE} + \mathcal{S}_{ENR}$, the multigrid method involving only $\mathcal{S}_{FE}$ and a preconditioned conjugate gradient method involving $\mathcal{S}_{ENR}$, to solve the linear system of the GFEM. Using the solution obtained from the iterative solver, we will also report an *aposteriori error estimator* of the energy norm of the error and comment on its effectivity. We will show that the angle between $\mathcal{S}_{FE}$ and $\mathcal{S}_{ENR}$ for a particular GFEM, known as the *stable GFEM* (SGFEM), is $\approx \pi/6$ for *various mesh configurations relative to the interface*, and thus it is robust with respect to accuracy, conditioning, and efficient iterative solution of the linear system.