A STUDY ON TIME DOMAIN BIEM WITH $\mathcal{H}$-MATRIX

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In a time domain BIEM(TD-BIEM), the boundary integral equation is discretized with a temporal and spatial interpolation function $M_T^m(\tau), M_S^j(y)$ as follow,

$$0 = \sum_{m=1}^{n} U_{n-m+1} q_m - \sum_{m=1}^{n} W_{n-m+1} u_m,$$

$$\{u_m\}_i := u(x^i, m\Delta t), \quad \{U_{m+1}\}_{ij} := \int \int \Gamma(x^i - y, \tau) M_S^j(y) M_T^m(\tau) d\tau dS,$$

$$\{q_m\}_i := \frac{\partial u}{\partial n}(x^i, m\Delta t), \quad \{W_{m+1}\}_{ij} := \int \int \frac{\partial \Gamma}{\partial n}(x^i - y, \tau) M_S^j(y) M_T^m(\tau) d\tau dS,$$

here, $x^i$ is the observation point and $\Delta t$ is the time increment. In scalar wave problems in 3D, the coefficient matrices $U_m, W_m$ are sparse since the fundamental solution $\Gamma(x, t) = \delta(t-|x|/c)$, here $c$ is the wave velocity. In the conventional TD-BIEM, one calculates the distance $r_{ij} = |x^i - y|$, $y \in \text{supp}\{M_S^j(y)\}$ and stores only the non-zero components of $U_m, W_m$ such that $m\Delta t \leq r_{ij}/c \leq m\Delta t - \text{supp}\{M_T^m(t)\}$ at every time steps. While the amount of calculation of $U_m, W_m$ is $O(N^2)$, the amount of calculation of $r_{ij}$ are $O(N^2 N_T)$, here, $N$ is the number of the boundary elements and $N_T$ is the number of time steps.

In this study, TD-BIEM with $\mathcal{H}$-matrix is considered. We divide the boundary into clusters $Q$ using a binary tree. We calculate the distance dist$\{Q^x, Q^y\}$ and only consider the submatrices of the hierarchical matrices $U^H_m, W^H_m$ such that $m\Delta t \leq \text{dist}\{Q^x, Q^y\}/c \leq m\Delta t - \text{supp}\{M_T^m(t)\}$ at every time steps. We also approximate the target submatrices which include non-zero components as low-rank matrices using ACA. The amount of calculation of dist$\{Q^x, Q^y\}$ and $U^H_m, W^H_m$ are $O((N \log N) N_T)$.

REFERENCES