11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20–25, 2014, Barcelona, Spain

A STUDY ON TIME DOMAIN BIEM WITH $\mathcal{H}$-MATRIX

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Key words: *time domain BIEM, $\mathcal{H}$-matrix, ACA, wave equation*

In a time domain BIEM(TD-BIEM), the boundary integral equation is discretized with a temporal and spatial interpolation function ${M_T}^m(\tau)$, ${M_S}^j(\boldsymbol{y})$ as follow,

$$0 = \sum_{m=1}^{n} \boldsymbol{U}_{n-m+1}\boldsymbol{q}_m - \sum_{m=1}^{n} \boldsymbol{W}_{n-m+1}\boldsymbol{u}_m,$$

$$\{\boldsymbol{u}_m\}_i := u(\boldsymbol{x}^i, m\Delta t), \quad \{\boldsymbol{U}_{m+1}\}_{ij} := \int\int \Gamma(\boldsymbol{x}^i - \boldsymbol{y}, \tau){M_S}^j(\boldsymbol{y}){M_T}^m(\tau)d\tau dS,$$

$$\{\boldsymbol{q}_m\}_i := \frac{\partial \boldsymbol{u}}{\partial n}(\boldsymbol{x}^i, m\Delta t), \quad \{\boldsymbol{W}_{m+1}\}_{ij} := \int\int \frac{\partial \Gamma}{\partial n_y}(\boldsymbol{x}^i - \boldsymbol{y}, \tau){M_S}^j(\boldsymbol{y}){M_T}^m(\tau)d\tau dS,$$

here, $\boldsymbol{x}^i$ is the observation point and Δt is the time increment. In scalar wave problems in 3D, the coefficient matrices $\boldsymbol{U}_m, \boldsymbol{W}_m$ are sparse since the fundamental solution $\Gamma(\boldsymbol{x}, t) = \frac{\delta(t-|\boldsymbol{x}|/c)}{4\pi|\boldsymbol{x}|}$, here c is the wave velocity. In the conventional TD-BIEM, one calculates the distance $r_{ij} = |\boldsymbol{x}^i - \boldsymbol{y}|,\ \boldsymbol{y} \in \mathrm{supp}\{{M_S}^j(\boldsymbol{y})\}$ and stores only the non-zero components of $\boldsymbol{U}_m, \boldsymbol{W}_m$ such that $m\Delta t \leq r_{ij}/c \leq m\Delta t - \mathrm{supp}\{{M_T}^m(t)\}$ at every time steps. While the amount of calculation of $\boldsymbol{U}_m, \boldsymbol{W}_m$ is $\mathcal{O}(N^2)$, the amount of calculation of r_{ij} are $\mathcal{O}(N^2 N_T)$, here, N is the number of the boundary elements and N_T is the number of time steps.

In this study, TD-BIEM with $\mathcal{H}$-matrix is considered. We divide the boundary into clusters Q using a binary tree. We calculate the distance $\mathrm{dist}\{Q^x, Q^y\}$ and only consider the submatrices of the hierarchical matrices $\boldsymbol{U}^H_m, \boldsymbol{W}^H_m$ such that $m\Delta t \leq \mathrm{dist}\{Q^x, Q^y\}/c \leq m\Delta t - \mathrm{supp}\{{M_T}^m(t)\}$ at every time steps. We also approximate the target submatrices which include non-zero components as low-rank matrices using ACA. The amount of calculation of $\mathrm{dist}\{Q^x, Q^y\}$ and $\boldsymbol{U}^H_m, \boldsymbol{W}^H_m$ are $\mathcal{O}((N \log N)N_T)$.

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