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A FAST DIRECT SOLVER FOR THE BOUNDARY ELEMENT METHOD WITH PMCHWT FORMULATION

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Boundary element method (BEM) is one of the numerical methods to solve boundary value problems of partial differential equations. The BEM is especially useful for wave scattering problems for unbounded domains since the numerical solution obtained by BEM automatically satisfies radiation condition, and also, discretisations only of the boundaries are required and those of the domain extending infinitely are not needed.

The computational complexity of the BEM is, however, either $O(N^3)$ with conventional direct solver or $O(N^2)$ with iterative solver since the coefficient matrices of the algebraic equations, denoted by A, are dense, where N is the dimension of A. To resolve this complexity issue, so called fast BEMs have been proposed. Among them, fast multipole methods (FMMs) [1, 2, 3] are the most widely used nowadays. Such FMMs can calculate a matrix vector product $\mathsf{A}\boldsymbol{x}$ for arbitrary vector $\boldsymbol{x}$ with the complexity of $O(N(\log N)^\alpha)$, ($\alpha = 0, 1$ or 2). With the help of iterative solvers, FMMs can be used as a fast method to solve linear equations obtained as discretised boundary integral equations. The FMMs are efficient as long as the number of iteration required by the solver is not so large. In the case that the coefficient matrix is ill-conditioned, even the FMMs cannot accelerate the BEM solver sufficiently.

Recently, to resolve the above issue, fast direct solvers for BEM have been proposed [4]. The computational cost with the direct solver is less affected by the condition of the matrix than that with the iterative solver. The method is based on the fact that the off-diagonal blocks of A are rank-deficient. The rank-deficient blocks are compressed by a variant of QR decomposition and the inverse of A is calculated via the compressed matrix representation.

The method is, so far, applied to boundary value problems of two-dimensional Laplace's equation, two-dimensional Helmholtz' equation in low frequency range [4], three-dimensional Helmholtz' equation [5]. Also, one can find an attempt to apply the method to transmission problems of Helmholtz' equation in two-dimensional periodic domain [6, 7].

In [6, 7], the integral equations used to formulate the transmission problems are a variant of Müller's formulation whose integral equation is of 2nd kind. Another possibility to formulate the transmission problems is the PMCHWT formulation whose integral equation is of 1st kind [8]. Since the PMCHWT formulation is widely used [8], it is worth studying the applicability of the fast direct solver to the boundary integral equation with PMCHWT formulation. In an oral presentation, we show the detailed formulation of the fast direct solver for PMCHWT-BEM with numerical examples which verify the efficiency of the proposed method.

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