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A HIGHER ORDER FLUX FOR MAGNETOHYDRODYNAMICS

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The equations of ideal Magnetohydrodynamics(MHD) have been frequently used to model many of the phenomena occurring in nature. Typical fields of study are space, heliospheric, magnetospheric and astrophysical phenomena.

Here, we present two AUSM-family fluxes originally developed for the Inviscid Euler Equations and extend them for use in MHD. The stability of these schemes is compared with other flux functions by evaluating the number of cases for which the flux function proves to be robust and does not produce divergent and unstable results.

Figures 1 and 2 indicate the robustness of these schemes for various 2-D standard test cases for 1^{st} order and 2^{nd} order accuracy, respectively.

	HLLE	HLLC	AUSM	AUSM⁺	SLAU2
Orszag-Tang Vortex	✓	✗	✓	✓	✓
Magnetized Blast	✓	✓	✓	✗	✓
Cloud-Shock Interaction	✓	✗	✗	✓	✓
Rotor 1	✓	✓	✗	✓	✓
Rotor 2	✓	✗	✗	✓	✓

Fig. 1: Table showing robustness of fluxes for 1^{st} order simulations

	HLLE	HLLC	AUSM	AUSM⁺	SLAU2
Orszag-Tang Vortex	✓	✗	✗	✗	✓
Magnetized Blast	✓	✓	✓	✗	✗
Cloud-Shock Interaction	✓	✗	✗	✓	✓
Rotor 1	✓	✓	✗	✓	✗
Rotor 2	✓	✗	✗	✓	✓

Fig. 2: Table showing robustness of fluxes for 2^{nd} order simulations

Further studies need to be conducted to study other aspects apart from the robustness of these schemes. There is also significant scope for modification of these new fluxes for further improvement.

Also, one of the most crucial aspects of MHD is the divergence-free evolution of magnetic fields. We use a divergence-free reconstruction technique by Balsara making the use of staggered grids to tackle this problem. The discretized divergence of the numerical solution of the magnetic field is calculated and tabulated as well to demonstrate the effectiveness of the staggered grid formulation. We also use a MUSCL-type limiter for achieving 2$^{\mathbf{nd}}$ order accuracy.

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