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Large SVD computations for analysis of inverse problems in geophysics

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SVD analysis is powerful tool to solve and analyse inverse problems in geophysics [1],[2]. For a large size of problem, computational resources significantly limit the computational time and memory usage of direct SVD algorithm. Partially it can be resolved by using HPC cluster parallelization algorithms. Alternatively, direct SVD approach can be replaced by using some limitation and simplify the input model [3],[4],[5]. Linearized inverse theories appear to be powerful tools, lending interesting insight into the physical interpretation of various methods of inversion and migration.

In this paper, we present SVD low-rank approximation algorithm. It can be used to make SVD analysis of the linearized problems namely inversing seismic data in Born approximation. The idea of the algorithm is based on looking for orthogonal vectors, which approximate subspace, spanned on major singular vectors with the largest singular values. High performance of proposed algorithm is based on the fast decreasing singular values of Born matrix and on the application of adaptive cross approximation (ACA) technique [6]. Performance of intermediate steps is improved by using BLAS and LAPACK components from Intel Math Kernel Library (Intel MKL) that is optimized for Intel architecture and parallelized via OpenMP [7].

To verify proposed algorithm, we compared obtained singular values and singular vector with the results provided by SVD algorithm from Intel MKL. Dependency approximation error of low-rank SVD, internal accuracy of low-rank approximation and output threshold of cropping the robust SVD was checked. The accuracy and threshold were varied from 10^{-3} to 10^{-9}. Results of the tests showed that the angle between subspaces spanned on singular vectors of robust SVD and Low-rank SVD is strongly correlated with internal accuracy and output threshold. In the worst case, the angle is about one degree. In the best case, it decreased down to 10^{-5}. The similar results were obtained for singular values error that varies from 10^{-2} to 10^{-9}.

To test performance we run the proposed algorithm on Born matrix with number of rows equal to 29 000, columns – 7 200. This size is large for computers with RAM of 8GB. Also as in the previous validation test, we compare obtained results with SVD results obtained with the use of Intel MKL.

Four modifications of Low-rank SVD were tested. They differ from each other by the type of low-rank approximation of internal matrix blocks: based on SVD, QR, ACA and blocked ACA approach. All these algorithms show performance gain: 1.5x (SVD), 3x (QR), 7x (ACA) and the fastest algorithm is Blocked ACA – up to 12x. Performance measurement was carried out on the Intel Core i7-3770K CPU 3.5 GHz, (Ivy Bridge). To make clear experiments we try to avoid impact of OMP parallelization all of MKL functions by switching off threading (set OMP_NUM_TREADS=1).

Due to the low-rank properties, the memory consumption of the algorithm should be less than of SVD in the robust arithmetic. Testing was done on the similar matrices like in performance tests. Results showed the performance gain up to 2x for all four modifications of proposed low-rank SVD algorithm.

The presented algorithm allows making SVD of large matrices, which cannot be decomposed by standard direct algorithms. Validation tests confirmed its high quality. Performance tests showed up to 12x speed up (on one thread) of proposed algorithm in comparison with SVD one from Intel MKL. Moreover, this algorithm has a large opportunity for parallelization both on shared memory systems (OMP parallelization) and on distributed ones (MPI parallelization).

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