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A VERY HIGHER ORDER CELL CENTERED FINITE VOLUME SCHEME FOR THE SIMULATION OF OIL-WATER DISPLACEMENTS IN PETROLEUM RESERVOIRS

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The numerical simulation of two-phase flows of oil and water in heterogeneous and anisotropic rock formations is a challenging task due to the difficulties associated to the modelling of such complex geological formations. In general, commercial petroleum reservoirs simulators use a linear Two Point Flux Approximation (TPFA) and a First Order Upwind Method (FOUM) to discretize the diffusive and the advective terms associated to the mathematical model that describes the fluid flow in petroleum reservoirs [1]. However, it is well known that the FOUM scheme presents excessive numerical diffusion leading to poor resolution of saturation/concentration fronts being also prone Grid Orientation Effects (GOE) particularly for flows with high mobility ratios [2] and in the last few years robust and efficient high-resolution methods have been proposed simulate the fluid flow in porous media [3,4]. Besides, for full permeability tensors and non K-orthogonal grids, the linear TPFA method is not consistent at all, presenting O(1) errors that do not vanish with grid refinement [3]. Under certain simplifying assumptions, the fluid flow in petroleum reservoirs can be described by an elliptic pressure equation and a non-linear hyperbolic saturation equation, which are solved by an Implicit Pressure Explicit Saturation (IMPES) procedure [4]. In this method the pressure equation is solved implicitly and the saturation equation is solved explicitly. In the present paper, in order to properly solve these equations, we adopt a full finite volume procedure. To discretize the elliptic pressure equation we use the Flux Continuous Finite Volume method with Full Pressure Support as proposed in [5] due to its robustness to handle heterogeneous and highly anisotropic porous media. On the other hand, to approximate the advective term that characterizes the hyperbolic saturation equation, we use the recently developed, Multidimensional Optimal Order Detection (MOOD) method [6]. In the MOOD scheme, which is an arbitrary higher order finite volume scheme adapted to general 2-D polygonal meshes (unstructured and non-orthogonal), within a time step, a first solution is evaluated using approximate numerical fluxes, computed from unlimited higher order local polynomial reconstructions (based, for instance, on a least square or spectral reconstruction), and then, through an iterative decremental process, the polynomial degree is reduced on control volumes where certain stability conditions are violated. In order to verify the accuracy and robustness of the proposed full finite volume strategy, we solve some benchmark problems found in literature.

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