

# CONTACT-IMPACT TREATMENT BASED ON THE BIPENALTY TECHNIQUE IN EXPLICIT TRANSIENT DYNAMICS

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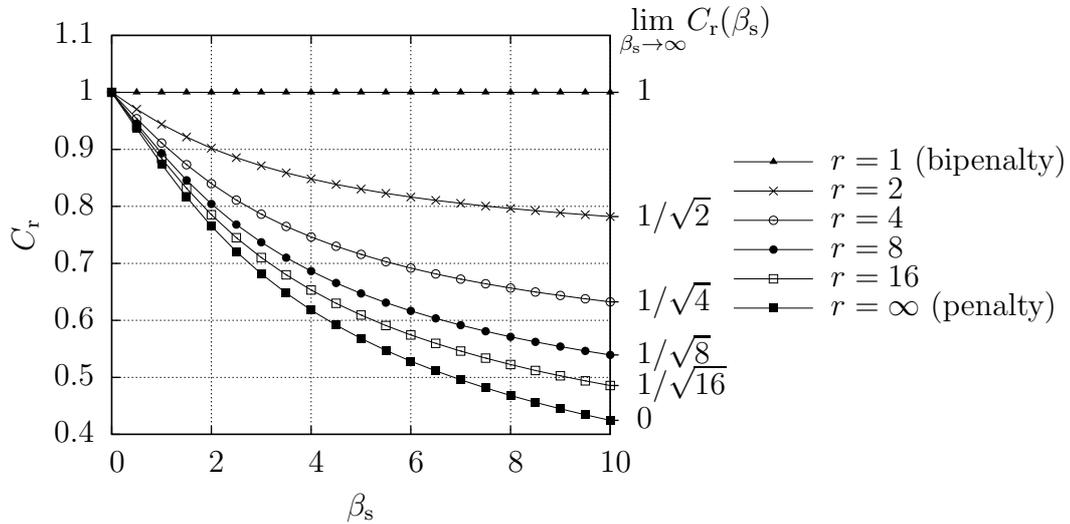
**Key words:** *contact-impact, bipenalty method, explicit transient dynamics*

In dynamic transient analysis, recent comprehensive studies have shown that the penalty method for the enforcement of contact constraints can be applied to both stiffness and mass matrix simultaneously. The aim of this bipenalty method is to find the optimum of the so-called critical penalty ratio (CPR) defined as the ratio of stiffness and mass penalty parameters that does not affect the maximum eigenfrequency  $\omega_{\max}$  of a system [1]. Hence, the critical time step  $\Delta t$  is also preserved for conditionally stable integration scheme because the linear stability theory establishes the upper bound of the time step as  $\Delta t \leq 2/\omega_{\max}$ .

However, there are no stability theorems for contact-impact problems. In this case the linear stability theory can be applied carefully. In practise, for example, the stability may be preserved by checking the energy balance during a nonlinear computation. In Reference [2] an upper bound for the stiffness penalty was derived. Furthermore, this estimation was generalized for the bipenalty method in Reference [3], which is described in more detail now.

Based on the solution of the eigenvalue problem of a simple dynamic system with two degrees of freedom the upper bound of the stable Courant number for the bipenalty method was obtained. The dependence of the Courant number  $C_r$  on the dimensionless stiffness penalty  $\beta_s$  is plotted in Figure, where the dimensionless penalty ratio  $r = \beta_s/2\beta_m$  is employed as the parameter ( $\beta_m$  is the dimensionless mass penalty). The curve for  $r \rightarrow \infty$  (i.e.  $\beta_m \rightarrow 0$ ) corresponds to the standard stiffness penalty method. It illustrates the main disadvantages of the standard stiffness penalty method: the Courant number  $C_r$  rapidly decrease with increasing dimensionless stiffness penalty  $\beta_s$ . On the other hand, the curve for  $r = 1$  confirms the existence of the CPR, for which the stable time step remains unchanged for an arbitrary value of the dimensionless stiffness penalty  $\beta_s$ . In addition, there are more curves in Figure for dimensionless penalty ratios  $r = 2, 4, 8$ , and 16. For

each of them, there are limits of the Courant number for  $\beta_s \rightarrow \infty$  on the right edge of the picture.



In this work, the bipenalty approach is applied to an explicit algorithm based on the pre-discretization penalty formulation [4]. The attention is focused on the stability properties of this algorithm using the derived upper bound of the stable Courant number. Several numerical examples are presented including the longitudinal impact of two thick plates, for which an analytical solution is available. In all the cases the superiority of the bipenalty method over the standard penalty method is demonstrated.

### Acknowledgements

Support by GAP101/12/2315 with institutional support RVO:61388998 is acknowledged.

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