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AN EXPLICIT STAGGERED SCHEME FOR THE COMPRESSIBLE EULER EQUATIONS

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The aim of the work presented here is to build numerical schemes for the compressible Euler equations that have the following properties: our scheme must preserve the positivity of the density and the internal energy, the integral of the total energy over the domain must remain constant and the scheme should naturally degenerate to common stable schemes for incompressible flows as the Mach number tends to zero (implicit version).

We present an explicit staggered scheme, extending the idea developed in [2]. Scalar variables are defined on a primal mesh, while vectorial variables are defined on a dual mesh centered around the edges of the primal mesh. For the third equation of the system, we choose the internal energy balance. This equation, like the mass balance, is resolved using a MUSCL interpolation for the convection terms. Thanks to some CFL conditions, the scheme preserves the positivity of the energy, the density and the pressure. We derive a discrete kinetic energy balance from the momentum and the mass balance equation, containing remainder terms, which correspond to the dissipation introduced by the numerical viscosity, and do not vanish (in a distribution sense) at the limit of vanishing time and space steps. Consequently we add a corrective source term in the internal energy equation in order to compensate the remainder terms of the discrete kinetic energy balance and we recover a discrete total energy balance.

Our scheme is shown to preserve the energy of the flow (*i.e.* the integral of the total energy over the computational domain), and keeps the velocity and the pressure constant across the 1-dimensional contact discontinuity. In addition, we prove its consistency, which means that the limit of a converging sequence of discrete solutions is necessarily a weak solution of the Euler equations.

In our numerical experiments, we tested several 1D and 2D Riemann problems. The scheme shows good accuracy near contact-waves and rarefactions. Shocks are sharply computed though oscillations are observed for strong ones. These oscillations are damped using artificial viscosity in the momentum equation, derived from [3]. The process does not affect the overall accuracy of the scheme nor its consistency.

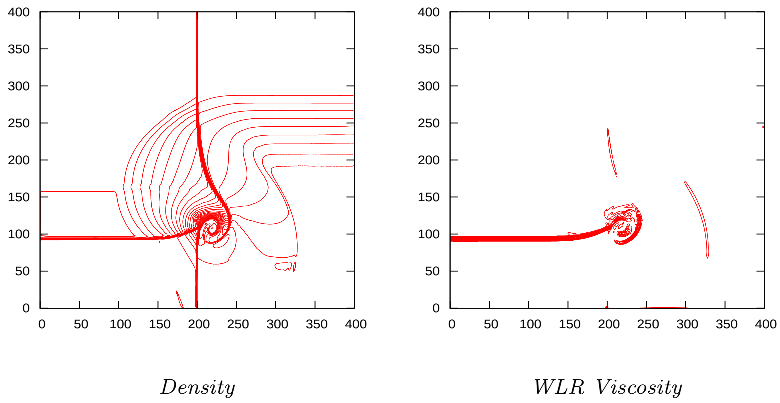

Figure 1: Riemann problem from configuration 17 in [4] (40 contours).

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