

# A FULLY 3D HYBRID NITSCHKE AND LEVEL SET METHODS FOR ELECTROHYDRODYNAMIC POTENTIAL FLOWS IN MOVING DOMAINS

August Johansson<sup>1</sup>, Maria L. Garzon<sup>2</sup> and James A. Sethian<sup>3</sup>

<sup>1</sup> Mathematics Department, Lawrence Berkeley National Laboratory, USA, ajohansson@lbl.gov

<sup>2</sup> Department of Applied Mathematics, University of Oviedo, Spain,  
maria@orion.ciencias.uniovi.es

<sup>3</sup> Department of Mathematics, UC Berkeley and Mathematics Department, Lawrence Berkeley  
National Laboratory, USA, sethian@math.berkeley.edu

**Key words:** *Nitsche finite element method, Fictitious Domain, Level Set Method, Potential Flow, Electrostatic Driven Flows, Charged Droplets*

A method for potential flow computations in moving and breaking domains was developed by Garzon and Sethian in a series of papers. The method uses the level set embedding techniques to establish a complete Eulerian formulation of the classical Lagrangian equations, and was approximated using a boundary integral formulation. The model has proved to be very robust to simulate various physical situations such as wave overturning and breaking [1]; the Taylor-Rayleigh instability of a fluid jet, [2]; droplet and bubble evolution in a two fluid system, [3], [4] ; and more recently electrical droplet deformation, [5]. The algorithm was developed assuming axisymmetrical geometries and due to the use of the boundary element method, an explicit mesh of the free boundary has to be constructed at each time step.

In this talk we will present a method that is based on the classical Nitsche Finite Element Method [6] which allows for unfitted meshes, *i.e.* the mesh does not have to fit the computational domain. These methods can be of arbitrary order and have general boundary conditions. However, very small volumes may occur at the boundary, particularly if the boundary is moving. This leads to breakdown of the finite element method since the bilinear form is no longer coercive, and numerical instabilities appear in the form of singular linear systems. To circumvent this we take use of Burman's ghost penalty method [7].

We will present results for full 3D electrohydrodynamical problems included breaking and pinching of droplets.

## REFERENCES

- [1] M. Garzon, D. Adalsteinsson, L. J. Gray, J. A. Sethian. A coupled level set-boundary integral method for moving boundary simulations. *Interfaces and Free Boundaries*. 7:277-302, 2005.
- [2] M. Garzon, L. J. Gray, J. A. Sethian. Numerical simulation of non-viscous liquid pinch-off using a coupled levelset-boundary integral method. *J. Comput. Phys.* 228:6079-6106, 2009.
- [3] M. Garzon, L. J. Gray, J. A. Sethian. Simulation of the droplet-to-bubble transition in a two-fluid system. *Phys. Rev. E*. 83:046318, 2011.
- [4] M. Garzon, L. J. Gray, J. A. Sethian. Axisymmetric boundary integral formulation for a two-fluid system. *Int. J. Numer. Meth. in Fluids* 69:1124-1134, 2012.
- [5] M. Garzon, L. J. Gray, J. A. Sethian. Numerical simulations of electrostatically driven jets from non-viscous droplets. *Phys. Rev. E*. submitted, 2013.
- [6] J. Nitsche. Über ein Variationsprinzip zur Lösung von Dirichlet-Problemen bei Verwendung von Teilräumen, die keinen Randbedingungen unterworfen sind. *Abh. Math. Sem. Hamburg*,36:9-15, 1971.
- [7] E. Burman. Ghost penalty. *Comptes Rendus Mathematique* 348: 1217-1220, 2010.