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NONPARAMETRIC MODELLING OF MULTI-STAGE ASSEMBLIES OF MISTUNED BLADED DISKS

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In the field of turbomachinery, various assumptions are usually made to model bladed assemblies, in particular, (1) cyclic symmetry property of a single rotor stage and (2) isolated, dynamically independent rotor stages. The first assumption enables to reduce drastically the required computational resources by considering one sector instead of the entire assembly to model and analyse the dynamic behaviour of the structure. However, even if the bladed disks are designed to be cyclic symmetric, small random blade-to-blade variations appear in practice due to manufacturing tolerances, irregularities in the material properties, wear during use, etc. These small variations, known as *mistuning*, disturb dramatically the dynamic behaviour of bladed disks (energy localization in only a few blades, amplification of the forced response). The second assumption requires the choice of boundary conditions in order to represent constraints normally imposed by adjacent stages, thus enabling the description of the "global" dynamic behaviour of a single rotor stage in a multi-stage structure. However, this assumption may not accurately describe the disk flexibility locally at the interstage boundaries, leading to an inconsistent representation of the interaction between families of disk- and blade-dominated modes. The mixture of blade and disk dominance (i.e., the disk-blade modal interaction) in veering regions is a critical factor to determine the sensitivity of a design to mistuning [1].

This work proposes a new combination of two modelling techniques, incorporating the advantages of both methods to model more precisely the mistuned response of multi-stage assemblies with reduced computational time. Laxalde and al. [2] have used the parametric modelling of uncertainties developed by S.-H. Lim and al. [3] in the case of multi-stage cyclic symmetric assemblies. Capiez and al. [4] have developed a nonparametric approach for uncertainties modelling of bladed disks. In this work, we combine the reduced-order modelling technique based on multi-stage cyclic symmetry [2] with the nonparametric approach for modelling uncertainties [4] in order to introduce the uncertainties directly in the modal space of the blades of multi-stage structures. Table 1 summarizes the characteristics of the different methods.

Laxalde [2]	**Capiez [4]**	**Proposed Method**
Modal	Physical	Modal
Multi-stages	Mono-stage	Multi-stages
Parametric	Nonparametric	Nonparametric

Table 1: Comparison between the existing and the proposed methods

The principle of the proposed method is to separate the global structural matrices defined in the modal space (for example the reduced stiffness matrix $\mathbf{K}_r$) into two contributions, thanks to a Craig-Bampton modal synthesis method for the blades:

$$\mathbf{K}_r = \underbrace{\mathbf{K}_r^0}_{\substack{Disk\ + \\ Static\ modes}} + \underbrace{\sum_{s=0}^{S-1} \sum_{n=0}^{N_s-1} \mathbf{P}^T \mathbf{\Lambda}_{(s,n)}\ \mathbf{P}}_{Internal\ modes}, \quad (1)$$

in which S is the number of stages and N_s the number of blades of stage s, $s \in [0, S-1]$. The first term in Equation (1) takes into account the contribution of the disk and the static modes of the blades. The second term represents the contribution of the internal vibration modes of the blades (the matrix $\mathbf{\Lambda}_{(s,n)}$ contains the modal stiffnesses of the retained internal vibration modes of each blade), which are projected in the modal space of the whole system by means of the projection matrix $\mathbf{P}$. Then we introduce the uncertainties in the modal space of the blades using the probability model for symmetric positive-definite real random matrices proposed by Soize [5] and applied by Capiez and al. [4] to cyclic symmetric structures. The $\mathbf{\Lambda}_{(s,n)}$ matrix in Equation (1) is replaced with a random matrix $\mathbf{\Lambda}'_{(s,n)}$.

We perform a numerical comparison between the three methods in terms of amplification factors of the forced response, based on a mono-stage and a multi-stage bladed structure.

REFERENCES

[1] R. Bladh, M. P. Castanier, C. Pierre. Effects of Multistage Coupling and Disk Flexibility on Mistuned Bladed Disk Dynamics. *Journal of Engineering for Gas Turbines and Power*, Vol. **125**, 121–130, 2003.

[2] D. Laxalde, C. Pierre. Modelling and analysis of multi-stage systems of mistuned bladed disks. *Computers and Structures*, Vol. **89**, 316–324, 2011.

[3] S.-H. Lim, R. Bladh, M. P. Castanier, C. Pierre. Compact, Generalized Component Mode Mistuning Representation for Modeling Bladed Disk Vibration. *AIAA Journal*, Vol. **45**, 2285–2298, 2007.

[4] E. Capiez-Lernout, C. Soize. Nonparametric modeling of random uncertainties for dynamic response of mistuned bladed-disks. *Journal of Engineering for Gas Turbines and Power-Transactions of the Asme*, Vol. **126**, 610–618, 2004.

[5] C. Soize. A nonparametric model of random uncertainties in linear structural dynamics. *Publication du LMA-CNRS*, Vol. **152**, 109–138, 1999.