

NUMERICAL MULTISCALE MODELLING OF SUPERCONDUCTING STRAND USING MINIMAL KINEMATIC BOUNDARY CONDITIONS (MKBC) PROCEDURE OF HOMOGENISATION

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In several former papers (for example [1] and [2]) the authors have estimated thermo-mechanical properties of superconducting, hierarchically structured strand using asymptotic homogenisation or self-consistent approach. In the present paper we propose to use a **minimal kinematic boundary conditions (MKBC) procedure of homogenisation** (see [3]). Although all these methods give only approximate results, a comparative discussion concerning the usefulness of the MKBC approach is possible. This is a goal of the paper.

We can distinguish three scales in superconducting strands: the strand (macro scale), the filament group (meso scale), and the single filament (micro scale). Using a suitable homogenisation procedure at each structural level the composite structure can be replaced with an equivalent homogeneous material endowed with effective constitutive characteristics. Of course, the inverse procedure called here “unsmearing” should be also defined since the local stress distribution at the level of microstructure is fundamental for prediction of yielding.

Classically, homogenisation is done by an analysis of the micro and meso cells of periodicity, according to one of the classical theories (asymptotic homogenisation, self-consistent approach). In this paper we contrast these methods with the numerical MKBC procedure. This approach is basing on the analysis of representative volume element (RVE) taken into consideration and discretised “as is”. A requirement of the Hill-Mandel definition of equivalence of homogeneous behaviour of the composite is handled directly, using Lagrange multipliers. These Lagrange multipliers are then used to calculate unknown, dual effective materials characteristics. The explicitly defined Dirichlet boundary conditions for the RVE are limited here to the minimal set which eliminates rigid motion. The resulting algorithm is simple and numerically efficient.

In this paper we use also our own method of numerical definition of yield surface for a fictitious homogenised body in the space of effective stresses, published earlier in [4].

The advantages and disadvantages of methods of homogenisation on the example of the finite element solution obtained for an idealised model of superconducting strand are discussed. We observe that the results of the models are qualitatively similar while the numerical efficiency of the models is different, with a favour for the MKBC approach.

The proposed paradigm does not constraint the model to be periodic, does not require also any artificial interpretation of the geometry of the original micro or meso structure. The problem of good RVE is to be solved instead. In the multiscale framework this problem is solved easily by taking the micro structure “as is”. As far as we know the MKBC procedure has not been employed for multiscale homogenization-localisation (unsmearing) modelling.

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