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# A COMPATIBLE DISCRETIZATION APPROACH FOR THE INCOMPRESSIBLE EULER EQUATIONS

**Andrea Natale*[1] and Marc Gerritsma[1]**

[1] Delft University of Technology,
Kluyverweg 1, 2629 HS Delft, The Netherlands,
e-mail: nichel27@gmail.com, M.I.Gerritsma@tudelft.nl

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The incompressible Euler equations represent a first simplified model for the mathematical description of incompressible flows. These equations possess a rich geometric structure which is responsible for their many properties, such as symmetries and invariant quantities. Conventional approaches generally ignore this structure and as a consequence are unable to represent exactly invariant quantities at the discrete level. Our work is meant to show how the geometric interpretation of the equations can be profitably used to develop a conservative discrete formulation of the problem in the 2D case. The scheme derived is based on concepts of exterior calculus and on the interpretation of physical quantities as differential forms. This view implicitly defines guidelines for the discretization of variables which preserve topological relations. More in particular one is able to express the usual gradient, divergence and curl operators with one single differential operator named the exterior derivative d. Then, for example, using a compatible discretization ensures that $\mathrm{d} \circ \mathrm{d} = 0$, which encapsulates the familiar vector calculus identities $curl \circ grad = 0$ and $div \circ curl = 0$, [1]. These properties are necessary but not sufficient to guarantee conservation of global quantities at the discrete level. We show that, in order to obtain this, one needs to define in an appropriate way discrete metric-dependent relations, in particular the Hodge star operator, to maintain the necessary symmetries in a finite dimensional setting. In particular, we use an approach similar to the one described in [2] where the discrete Hodge operator is defined be means of the integral pairing of differential forms. This eventually leads to exact conservation of important invariants, such as kinetic energy, total vorticity and enstrophy.

## REFERENCES

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