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A FAST DIRECT SOLVER FOR ONE PERIODIC BOUNDARY VALUE PROBLEMS FOR HELMHOLTZ' EQUATION IN 2D

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Wave scattering problems for periodic structures are of interest since they have many applications in engineering and science, e.g. photonic crystals, metamaterials, etc. Boundary integral equation methods are expected to provide effective tools for such problems because they are considered suitable for wave applications. In periodic problems one usually obtains fairly large, but not huge problems because the periodicity assumption reduces the sizes of the problems although the structures under consideration can be complicated. Fast direct solvers are considered attractive in such problems.

As a matter of fact, several groups have already developed fast direct solvers for periodic wave problems[2, 3] based on approaches developed, e.g., by Martinsson and Rokhlin[1]. In the proposed presentation we discuss another fast direct solver for periodic problems developed from a different point of view. More specifically, we consider 1-periodic transmission problems for Helmholtz' equation in 2D. We use the periodised Müller formulation in which the periodic Green's function is used. The Kummer transformation[6] is utilised to decompose the Green's function into a sum of a quickly decaying part in the real space and another having a similar behaviour in the wavenumber (Fourier) space. The discretised integral operators corresponding to these parts are written separately as hierarchical matrices [4] with the help of the standard ACA for the former part and a semi-analytical approach for the latter. The hierarchical sum of these matrices is LU decomposed with the help of the H-matrix arithmetic.

We here present a simple numerical example. The unit cell consists of one circular scatterer having the radius of 0.49, while the period in the periodic direction x_1 is 1. We assume that the permeability is equal to 1 in all domains, the frequency is 2π, and the permittivity constants for the exterior and interiar are 30 and 20, respectively. Hence the wave numbers

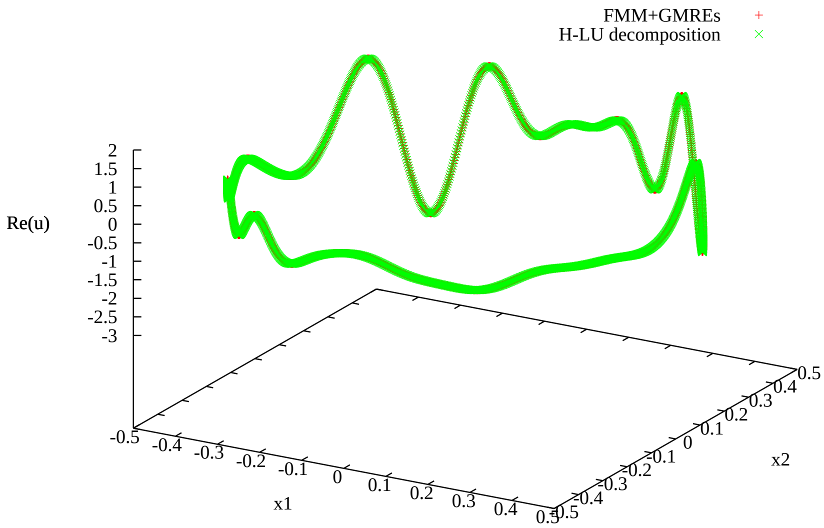

Figure 1: Comparison of the results obtained with the fast direct solver and pFMM.

for the exterior and interior are 34.4 and 28.1, respectively. The number of unkowns is 4000. This structure is subjected to a plane incident wave whose incident angle is 1 (rad). Fig. 1 compares the real parts of the solutions obtained with the proposed approach and the periodic FMM using the PMCHWT formulation[5] and GMRES. From Fig. 1, we see that the results obtained with the proposed approach and those with periodic FMM agree well. We plan to present more numerical examples in the proposed presentation.

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