

## EFFICIENT OPTIMIZATION ALGORITHMS FOR OPTIMAL CONTROL OF TURBULENT FLOWS

Cornelia C. Nita<sup>1\*</sup>, Stefan Vandewalle<sup>2</sup> and Johan Meyers<sup>1a</sup>

<sup>1</sup> Department of Mechanical Engineering, KU Leuven, Celestijnenlaan 300A, B3001 Leuven, Belgium, \*cornelia.nita@kuleuven.be, <sup>a</sup>johan.meyers@kuleuven.be,

<sup>2</sup> Department of Computer Science, KU Leuven, Celestijnenlaan 200A, B3001 Leuven, Belgium, stefan.vandewalle@cs.kuleuven.be

**Key words:** *Direct Numerical Simulation, Turbulent channel flow, Continuous adjoint method, Distributed optimal control, Conjugate gradient, Damped limited memory BFGS.*

The use of PDE-constrained optimization techniques in combination with transient three-dimensional turbulent flow simulations such as direct numerical simulation or large eddy simulation, requires high computational costs and storage. In this context, various procedures have been proposed and implemented that aim at reducing the cost of function evaluations (requiring expensive flow simulations), and storage requirements [5]. In the current study, we focus on the efficiency of gradient-based optimization algorithms, and investigate the number of simulation and adjoint simulations that are required per unit of cost-function improvement. Different algorithms are considered, i.e., two versions of a nonlinear conjugate gradient algorithm (CG) [2,3], a limited memory quasi-Newton method (L-BFGS) [1], a damped L-BFGS method [1], and truncated-Newton(TN) methods [4]. The comparative tests between L-BFGS and the truncated Newton methods are examined on an extended Rosenbrock function. Next to that, the L-BFGS, the damped L-BFGS and the CG methods are applied to an unconstrained distributed control problem that consists of the optimization of a distributed force in direct numerical simulations of a turbulent channel flow. The primal problem is governed by the Navier–Stokes equations where a distributed control is applied as an interior volume force on the right-hand side of the momentum equation. Numerical investigations are elaborated for the test case where the control is active only in a part of the simulation volume ( $\Omega_1$ ). The objective of interest in our optimization problem is to find the state variables (velocity) and the design parameters (volume force) so as to minimize:

$$\mathcal{J}(f, u) = - \int_0^T \int_{\Omega_1} f(z, T)u(z, T)d\Omega_1 dt. \quad (1)$$

This cost function maximizes power extraction by the control forces  $f$  in the turbulent channel flow over a time horizon  $T$ . The gradient of the cost functional  $\mathcal{J}$  Eq. (1)

is determined using the adjoint Navier–Stokes equations that are derived from the state system using the Lagrangian approach. The forward and adjoint equations are discretized using a mixed pseudo-spectral finite difference code and are integrated in time by a fourth order explicit Runge-Kutta scheme [1,3]. Firstly, the total number of function and gradient evaluations per iteration is studied for two line search procedures in the limited-memory BFGS method on an extended Rosenbrock function. For the same test case a Truncated Newton technique is performed and compared against the L-BFGS results (Figure 2a). Using insights from this test case, the nonlinear conjugate gradient Polak-Ribière (PRP) [3] and Dai-Yuan (DY) methods [2], L-BFGS and a damped version of L-BFGS are further applied for the optimization of a turbulent channel flow. We observe that the damped version of the L-BFGS is the most efficient method in terms of function and gradient evaluations (Figure 2b) with a faster decrease of the cost functional per iteration. More detailed results and discussion will be presented in the full paper.

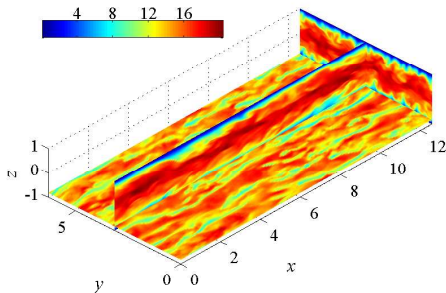


Figure 1. Contours of an instantaneous velocity field in a turbulent channel flow with a partial distributed force on  $z = [-0.8, -0.4]$ , Reynolds number  $Re_\tau = 180$ .

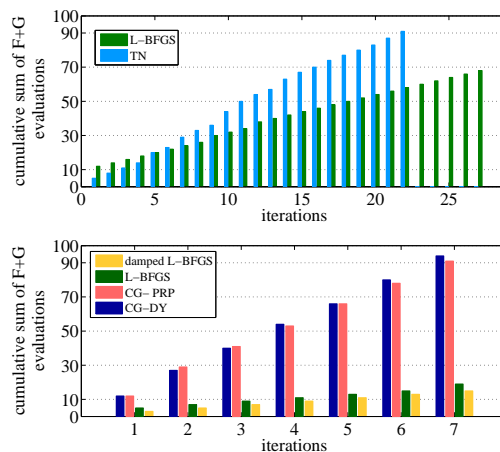


Figure 2. (a) L-BFGS & TN tested on Rosenbrock test function; (b) Algorithms tested on a turbulent channel flow case.

## REFERENCES

- [1] H. Badreddine, S. Vandewalle, J. Meyers. Sequential quadratic programming (SQP) for optimal control in direct numerical simulation of turbulent flows. *J. Comput. Phys.*, Vol. **256**, 1–16, 2014.
- [2] Y.H. Dai, Y. Yuan. A nonlinear conjugate method with a strong global convergence property. *SIAM J. Optim.*, Vol. **10**, 177–182, 1999.
- [3] S. Delport, M. Baelmans, J. Meyers. Constrained optimization of turbulent mixing-layer evolution. *J. Turbul.*, Vol. **10**, 1–28, 2009.
- [4] S.G. Nash. A survey of truncated-Newton methods. *J. Comput. Appl. Mat.*, Vol **124**, 45–59, 2000.
- [5] D. Thevenin, G. Janiga. *Optimization and Computational Fluid Dynamics*. Springer, 2008.