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SMOOTH ESTIMATE OF THE TRUNCATION ERROR FOR UNSTRUCTURED MESH FINITE VOLUME METHODS

Mahkame Sharbatdar[1] and Carl Ollivier-Gooch[2]

[1] PhD candidate, Mechanical Engineering Department, University of British Columbia, Vancouver, Canada, mahkame@interchange.ubc.ca

[2] Professor, Mechanical Engineering Department, University of British Columbia, Vancouver, Canada, cfog@mech.ubc.ca

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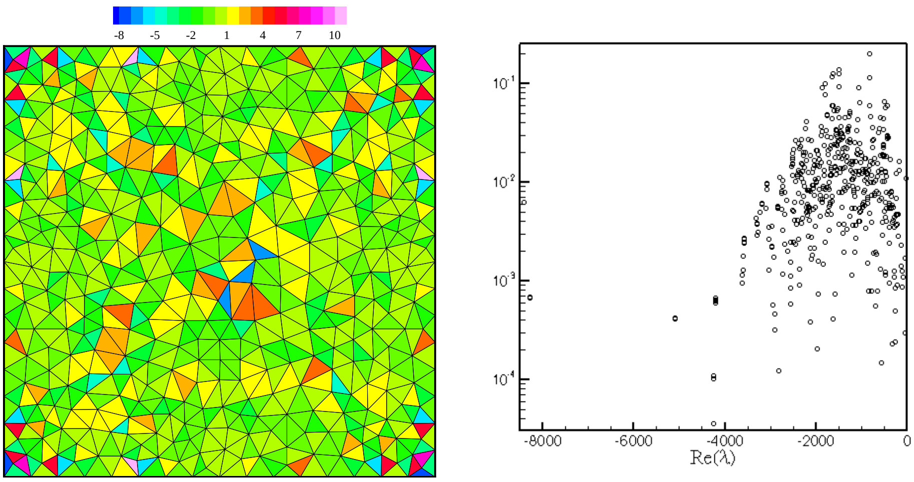
Studying the impact of unstructured mesh properties on the accuracy of finite volume simulations is an open area of research in CFD, although unstructured mesh finite volume methods are in widespread use. Historically, the main tool for quantifying solution accuracy has been grid refinement studies for which the cost is prohibitive for real-world problems. Meeting this need for error quantification for industrial scale CFD requires a new paradigm with error quantification capabilities. Most commercial CFD software handles real-world problem geometry using unstructured meshes, for which accuracy issues are not as well understood as for structured meshes. Developing methods for error quantification in a unified framework compatible with second-order, and ultimately higher-order, finite volume methods will be described here.

One measure of the accuracy of the scheme is the truncation error for which analysis for unstructured mesh schemes has lagged behind. Several researchers, including Jalali and Ollivier-Gooch [1], have demonstrated that the truncation error for unstructured finite volume schemes is asymptotically larger than the discretization error. This behavior is in contrast with the structured mesh case where the truncation error has the same asymptotic order of accuracy as the discretization error. Another feature of the truncation error for unstructured mesh schemes is its noisy appearance, caused by the discontinuous jump of the coefficients of the terms in the Taylor series expansion of the error from one control volume to another. This is in contrast with the structured mesh schemes with smooth truncation error. These two features of the truncation error for unstructured mesh finite volume schemes are related to each other by the eigensystem of the discrete problem.

Error quantification hinges on the ability to accurately estimate the local truncation error. The truncation error estimate can be used directly to improve the solution through defect correction, to estimate the error in an output quantity of engineering interest, and to estimate the discretization error by the error transport equation [2]. Some experiments were performed for a model Poisson problem in a square domain with unstructured mesh solved to second-order accuracy by the finite volume method for both defect correction and output error estimation and our results are not as they good as they are when the problem is solved on structured mesh. By implementing defect correction on a structured mesh, the discretization error of the corrected solution is asymptotically smaller than the error of the uncorrected case one; however, asymptotically faster convergence is not obtained when an unstructured mesh is used. Although the error in computing an output functional on an unstructured mesh is reduced by using a corrected functional, which is based on truncation error, the convergence of error is not as fast as expected. Detailed investigation of our results have shown that the rough modes dominate the behavior of the truncation error on unstructured meshes and this is the fundamental reason of poor convergence of defect correction and output error estimation once unstructured mesh is used. The truncation error and the weights in its decomposition for the Poisson problem described above is depicted in the figure shown here. The leftmost modes are corresponding to these rough modes. Since eigendecomposition is done for this problem, it is easy to distinguish these rough modes and then filter them out to obtain a smoother estimate of the truncation error. The goal of this paper is to smooth the truncation error and then use the smoothed estimation of the truncation error for defect correction and output error estimation.



(a) Truncation error (b) Weights in Eigendecomposition

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