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TRANSONIC COMBUSTION: STEADY AND UNSTEADY POTENTIAL MODELS

William E. Tavernetti[1] **and Mohamed M. Hafez**[2]

[1] Graduate Student, Department of Mathematics, University of California, One Shields Ave. Davis, CA 95616, etavernetti@math.ucdavis.edu

[2] Professor, Department of Mechanical Engineering, University of California, One Shields Ave. Davis, CA 95616, mhafez@ucdavis.edu

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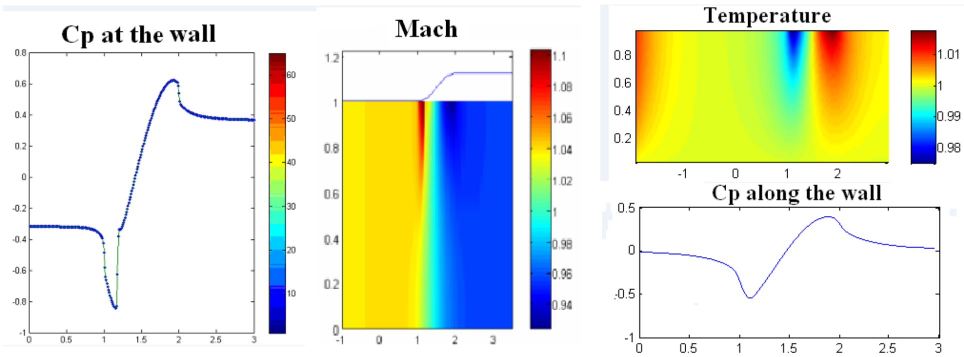
1 Summary

Transonic conditions in reactive flows occur in jet engines, power generation systems, missiles, turbo-machinery, industrial reactors and many more. Theoretical studies of combustion have mostly focused on low speed deflagrations as well as pure supersonic detonations; however, transonic conditions, have not been as widely studied. For transonic flow the governing equations are of mixed type, elliptic and hyperbolic, which in addition to the stiffness introduced by a reaction time scale, presents several numerical difficulties. The small disturbance approach is limited to small heat release, i.e. diluted premixtures, and small area variation. According to [2] the precise limitations are upstream flows with Mach number between 0.75 and 1.2, premixtures with reactant mass fractions up to 0.1, and channel cross-sectional area variation of 15 percent or less. This study has two primary goals: (1) to make a comparative study of the small disturbance and full potential models for steady reactive cases, and, (2) present an unsteady compressible reactive full potential model.

Following a classical approach using a small disturbance expansion around a uniform steady state (V_∞=1) gives the reactive transonic small disturbance model:

$$
\begin{aligned}
(1-M_\infty^2-(\gamma+1)M_\infty^2\varphi_x)\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2} &= \beta K_{\tilde{V}} A\tilde{Y}e^{-\theta/(1+\tilde{T})} \\
\tilde{T}=-(\gamma-1)\varphi_x, \qquad Y_x &= A\tilde{Y}e^{-\theta/(1+\tilde{T})}
\end{aligned}
\tag{1}
$$

Where: $\tilde{u}=\varphi_x$, $\tilde{v}=\varphi_y$, A, β, $K_{\tilde{V}}$, and θ are constant reaction control parameters. Non-reactive and reactive validation test cases are shown here:



(Non-Reactive)

(Reactive)

The unsteady compressible reactive full potential model is given by:

$$\begin{aligned}(\rho\phi_x)_x + (\rho\phi_y)_y + \rho_t &= 0\\ (\rho)_{tt} - (p)_{xx} &= (\rho u^2)_{xx} + (\rho uv)_{yx} + \epsilon(\rho)_{xxt} - \epsilon(\rho u)_{xxx}\\ (\rho Y)_t + (\rho uY)_x + (\rho vY)_y &= -D_a\rho Y e^{-\theta/T}\\ (\rho T)_t + (\rho uT)_x + (\rho vT)_y &= \frac{\partial p}{\partial t} + \beta D_a \rho Y e^{-\theta/T}\\ p &= \rho T\frac{\gamma-1}{\gamma}\end{aligned} \tag{2}$$

To solve this problem we use a conservative mixed upwind/centered scheme for the potential equation [4]. Non reactive steady and unsteady validations have been successful.

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