

A HIGHLY PARALLEL CODE FOR STRONGLY COUPLED FLUID-TRANSPORT EQUATIONS

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Many flow problems deal with transport of matter and/or heat. This constitutes a challenging multiphysics problem if the transported entity also influences the flow, leading to a two-way coupling. From a computing efficiency view point, it is best to treat the associated equations in a coupled manner [5]. Employing a domain decomposition approach, all the unknowns related to one domain should be in the memory of the node which treats that part and communication should be avoided as much as possible during the construction of the right-hand side, the construction of the Jacobian matrix and the solution process. Along this line we developed a finite volume package FVM and a solver HYMLS, both based on elements of the EPETRA-package (available within Trilinos (see <http://trilinos.sandia.gov/>)).

For this first version, we have chosen to use structured grids and finite volume discretizations. In fact for the incompressible Navier-Stokes equation, considered until now, we use the C-grid staggering. Our implementation is matrix oriented instead of function oriented. Before computation we compute and store all stencils needed in the code. From the nonlinear terms, which are here bilinear forms, we store the constituting operators also in the form of stencils. The Jacobian matrix and the right-hand side are now constructed from products of stencils and vectors. To solve the nonlinear equations we use the NOX package with our in house developed package HYMLS to solve the systems. HYMLS is a linear system solver for steady state incompressible Navier-Stokes equations coupled to transport equations in 2 and 3D [1, 2, 3]. We constructed recently a multilevel variant of it, which makes it possible to solve 3D problems of over 10 million unknowns

quickly on a parallel computer. The behavior of the method is very much like that of multigrid methods. In fact one could see it as the father of the multigrid method. The solver is very robust. For the problem described in [4], it allowed a quick increase in the Reynolds number to get into the interesting region around $Re=2000$. Here we will show the performance of the method on the Rayleigh-Bénard convection in a cube, with six no-slip walls [6].

To study the stability of the solutions we determine the eigenvalues using the ANASAZI-package, which contains a generalized version of the Arnoldi method. Also here we employ HYMLS to solve the linear systems resulting from a Cayley transform of the generalized eigenvalue problem. In the talk we will give a more detailed explanation of the used algorithms.

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