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A HIGHLY PARALLEL BIFURCATION ANALYSIS TOOL FOR FLUID FLOW PROBLEMS

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It goes without saying that bifurcation analysis of dynamical systems arising from partial differential equations needs efficient parallel code [5]. We built such a code on elements of the EPETRA-package available within Trilinos (see http://trilinos.sandia.gov/). The main idea is as follows. Both the construction of the matrix and the right-hand side is performed in parallel using a domain decomposition approach. The solution process exploits the same data layout, which keeps data movement between processors low. We chose to use structured grids and finite volume discretizations. For the incompressible Navier-Stokes equation we use the staggered C-grid. Our implementation is matrix oriented instead of function oriented. Before computation we compute and store all stencils needed in the code. Even the nonlinear terms, which are in fact bilinear forms, can be expressed as a combination of stencils. The Jacobian matrix and the right-hand side are now constructed from products from stencils and vectors. For the continuation process we use LOCA which calls HYMLS to solve the systems.

HYMLS is a linear system solver for steady state incompressible Navier Stokes equations in 2 and 3D [1, 2, 3]. We constructed recently a multilevel variant of it, which makes it possible to solve 3D problems of over 10 million unknowns quickly on a parallel computer. The solver is very robust and allows a quick increase in the Reynolds number to get into the interesting region where the first Hopf bifurcation is occurring in a lid-driven cavity (around Re=8000 in 2D and around 2000 in 3D) (see also [4]). In the neighborhood of this critical number we compute the most critical eigenvalues using the ANASAZI-package, which contains a generalized version of the Arnoldi method. Also here we employ HYMLS to solve the linear systems resulting from a Cayley transform of the generalized eigenvalue problem.

In the presentation we will give a more detailed explanation of the used algorithms and show the break up of the computation time over the various parts of the algorithm.

REFERENCES

[1] A.C. de Niet and F.W. Wubs. Numerically stable LDL^T factorization of F-type saddle point matrices. *IMA Journal of Numerical Analysis*, **29**(1), 208-234, 2009.

[2] F.W. Wubs and J. Thies, A robust two-level incomplete factorization for (Navier-)Stokes saddle point matrices, *SIAM J. Matrix Anal. Appl.*, **32**, 1475 - 1499, 2011.

[3] Jonas Thies and Fred Wubs, Design of a Parallel Hybrid Direct/Iterative Solver for CFD Problems. *Proceedings 2011 Seventh IEEE International Conference on eScience*, 5-8 December 2011, Stockholm, Sweden, pp. 387 -394, 2011.

[4] Yuri Feldman and Alexander Yu. Gelfgat, Oscillatory instability of a three-dimensional lid-driven flow in a cube, *Phys. Fluids*, **22**, 2010.

[5] H.A. Dijkstra et al., Numerical Bifurcation Methods and their Application to Fluid Dynamics: Analysis beyond Simulation, *Communications in Computational Physics (CiCP)*, **15**(1), 1-45, 2014.