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ISOTROPIC PROPERTIES OF LAX-WENDROFF METHODS, WITH APPLICATION TO LAGRANGIAN HYDROCODES

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Lagrangian calculations of fluid motion are preferred when there is a requirement to avoid numerical diffusion of different materials, but they frequently suffer from mesh instabilities that can lead to premature termination of the calculations. This paper will present studies of a closely related, but simpler situation. This is the solution of the linear acoustic system; we attempt to calculate, on a square grid, flows that should have radial symmetry.

The family of schemes considered correspond to cell-centered hydrocodes, with numerical fluxes that may be located either on the edges of cells or at their vertices, or in some combination determined by a parameter. There are four such parameters. Two of them are related to the evaluation of the first-derivatives, and two more to the second derivatives. The time-stepping is handled by a Lax-Wendroff procedure but the analysis applies to other choices. We study both first- and second-order versions, because we intend to use a flux-corrected transport strategy to control numerical overshoots[1], although our methodology is a little different from the one described there. For the low-order fluxes in the FCT method there are two more parameters that control the dissipation applied to the pressure and the velocities.

For the second-order schemes, we conduct a dispersion analysis to make the errors as isotropic as possible. Two of the parameterss must then be chosen so that the velocity fluxes must be evaluated at cell vertices, as is also required to preserve discrete vorticity[2]. The choices for the remaining parameters require a compromise between isotropy and stability.

Among the low-order schemes, we seek one that combines isotropy with freedom from spurious features. Surprisingly, the scheme that corresponds to first-order upwinding, and would therefore be optimal in one dimension, is rather poor from the standpoint of isotropy. It produces strong spurious oscillations for waves propagating diagonally across the grid. We are able to improve considerably on this behavior but only so far by empirical testing.

Improving the accuracy to third-order would of course require a larger stencil than the nine points that are typical for Lax-Wendroff. One way to achieve this is to take information from a provisional update to revise the estimates for the quantities that drive changes to the solution, such as pressure gradients and vorticity. These revised estimates also provide information about the local smoothness of the solution and can be used to motivate a limiter that preserves the isotropic properties. A separate paper on the limiter strategy has been submitted to another minisymposium.

By the time of the conference the methods will have been tested in the Lagrangian context.

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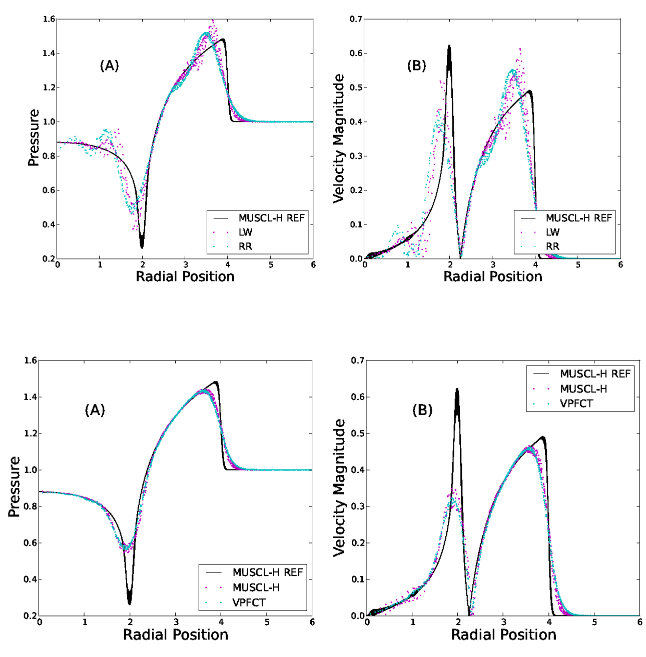

Figure 1. Scatter plots of a radially symmetric wave propagation problem solved on a 100x100 square grid. At top the unlimitedl Lax-Wendroff method before and after preserving vorticity; at bottom after imposing symmetry and limiting.