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# ON LINEARIZATION OF NONLINEAR DYNAMIC SYSTEMS DESCRIBED BY STATE-DEPENDENT-PARAMETER (SDP) DISCRETE-TIME MODEL

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**Introduction:** The description of nonlinear dynamic system using State-Dependent-Parameter (SDP) models can be found in earlier publications [1]. However, its practical development is of more recent origin, see for example [2] and the references therein. The nonlinear system is modelled using a quasi-linear model structure, in which the parameters are dependent on other variables in the system [2]. This SDP formulation is used for prediction and/or control [3]. Since not all feasible SDP models are controllable, the present work presents the exact linearization which brings the nonlinear SDP model back to controllable state [4].

**Nonlinear SDP model:** The discrete-time incremental form of an SDP model takes the form

$$y_k = -\sum_{i=1}^{n} a_{i,k}\, y_{k-i} + \sum_{j=1}^{m} b_{j,k}\, u_{k-j} \tag{1}$$

where $u_k$ and $y_k$ are the current system input and output respectively. However, the model parameters $a_{i,k}$ $(\forall 1 \le i \le n)$ and $b_{j,k}$ $(\forall 1 \le j \le m)$ are functions of the system states.

**Exact Linearization:** Exact linearization approach of discrete-time SDP-TF models has been developed by Shaban [4], for which the relative degree term $\rho$ has been introduced. This term helps to properly construct the SDP model, such that the sample delay $\delta$ equals its relative degree $\rho$ [4]. It is always possible to guarantee the parity between $\rho$ and $\delta$ of any nonlinear SDP models, i.e. $\rho = \delta$, by the proper arrangement of its incremental form (1), such that the regressors $y_{k-i}$ and $u_{k-j}$ are always lagged by equal or lesser steps than their corresponding time-variant parameters $a_{i,k}$ and $b_{j,k}$. In order to develop the linearized model for the nonlinear SDP model (1), the coordinate $U_{k-(\rho-1)}$ is introduced such that [4]

$$U_{k-(\rho-1)} = \Delta y_k - \sum_{i=2}^{n} y_{k-(i-1)} \tag{2}$$

for which $\Delta$ is the step forward difference operator, i.e. $\Delta y_k = y_{k+1} - y_k$, and $U_k$ is the linearized transitional input. The linearized model can now be constructed as

$$\Delta y_k = \sum_{i=2}^{n} y_{k-(i-1)} + U_{k-(\rho-1)} \tag{3}$$

The expansion of (3) with step-behind leads to the new linearized model in incremental form

$$y_k = y_{k-1} + y_{k-2} + \ldots + y_{k-n} + U_{k-\rho} \tag{4}$$

This is a linear system with unity parameters at all input and output terms.

The mapping between the transitional linearized input $U_k$ and the actual system input $u_k$ can be obtained from the $(\rho-1)^{\text{th}}$ difference of transformation (2) as

$$\Delta^{\rho-1}U_{k-(\rho-1)} = \Delta^{\rho} y_k - \sum_{i=2}^{n} \Delta^{\rho-1} y_{k-(i-1)} \tag{5}$$

**DC Motor Demonstrator:**
The model of a mechatronic system, see Figure (1), driven by A DC motor can be given as

$$y_k = -a_{1,k}\, y_{k-1} + b_{4,k}\, u_{k-4} \tag{6}$$

For which the state dependent parameters are given as

$$\begin{aligned} a_{1,k} &= -0.2085\times10^{-6}\, u_{k-4}^2 - 0.7224 \\ b_{4,k} &= -0.003425\times10^{-6}\, u_{k-4}^2 + 0.005869 \end{aligned} \tag{7}$$

According to the steps of linearization, the linearized model takes the form of

$$y_k = y_{k-1} + U_{k-4} \tag{8}$$

The PIP controller can now be implemented to the linearized model (4), see Figure (2), for which the controller is examined at two different levels of set points. The controller proves zero steady state error at both selected levels of set points.

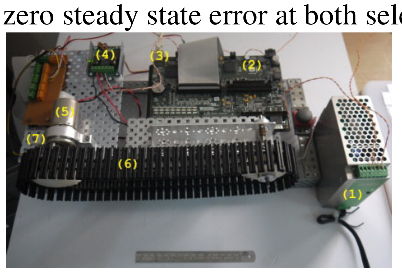


Figure 1 The mechatronic system used for evaluating the PIP control

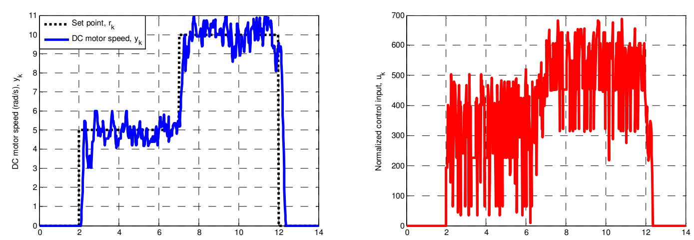


Figure 2 The implementation of PIP control based on the exact linearization method, at two levels of set points

**Conclusion:** This paper develops an exact linearization approach for a wide range of nonlinear systems, which based on the SDP models. Necessary and sufficient conditions are given such that the nonlinear SDP systems are equivalent to a controllable linear system. This analysis suggests that the exact linearization of SDP models is a straightforward approach.

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