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POSITIVITY PRESERVATION PROPERTY OF CELL-CENTERED LAGRANGIAN SCHEMES AND EXTENSION TO HIGH-ORDERS OF ACCURACY

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One of the main issue in the field of numerical schemes is to ally robustness with accuracy. And considering gas dynamics, at high Mach numbers or for flows near vacuum numerical approximations may generate negative density or pressure, which may lead to nonlinear instability and crash of the code. This phenomenon is even more critical using a Lagrangian formalism, the cells moving and being deformed during the calculation. In this talk, we first demonstrate in the two-dimensional case the positivity preservation property on the density, pressure and internal energy of first-order finite volume cell-centered Lagrangian schemes as GLACE [1] and EUCCLHYD [3], on generic polygonal meshes. This analysis enables us to derive a time step condition ensuring the desired positivity property as well as a production of discrete entropy and a L_1 stability of the specific volume and total energy over the domain. Adapting the work presented in [4, 2] to the cell-centered Lagrangian frame, this positivity study is then extended to high-orders of accuracy firstly using second-order MUSCL type schemes on moving mesh, and then high-order discontinuous Galerkin methods using a total Lagrangian formalism on general unstructured curvilinear grids. New time step constraints are then obtained, and a proper limitation is also needed. Through this new procedure, the scheme robustness is highly improved and hence new problems can been tackled. Numerical results are provided to demonstrate the effectiveness of these methods.

REFERENCES

[1] G. Carré, S. Delpino, B. Desprès and E. Labourasse, *A cell-centered Lagrangian hydrodynamics scheme in arbitrary dimension.* J. Comp. Phys., 228:5160-5183, 2009.

[2] J. Cheng and C.-W. Shu, *Positivity-preserving Lagrangian scheme for multi-material compressible flow.* J. Comp. Phys., 257:143-168, 2014.

[3] P.-H. Maire, R. Abgrall, J. Breil and J. Ovadia, *A cell-centered Lagrangian scheme for two-dimensional compressible flow problems.* SIAM J. Sci. Comp., 29:1781-1824, 2007.

[4] X. Zhang, Y. Xia and C.-W. Shu, *Maximum-principle-satisfying and positivity-preserving high order discontinuous Galerkin schemes for conservation laws on triangular meshes.* J. Sci. Comp., 50:29-62, 2012.