

# A NEW FRAMEWORK FOR STOCHASTIC ANALYSIS IN LARGE SCALE SIMULATIONS BASED ON GOAL-ORIENTED PROBABILITY DENSITY FUNCTION METHODS

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Determining the statistical properties of large-scale stochastic dynamical systems is a problem of major interest in many areas of science and engineering. Even with recent theoretical and computational advances, no broadly applicable technique has yet been developed for dealing with the challenging problems of high dimensionality, model uncertainty, lack of regularity and random frequencies. In this talk we present a new framework for stochastic analysis in large scale simulations based on goal-oriented probability density function (PDF) methods. The key idea stems from techniques of irreversible statistical mechanics, in particular the Nakajima-Zwanzig-Mori formalism, and it relies on deriving evolution equations for the PDF of low-dimensional quantities of interest, e.g., functionals of the solution to systems of stochastic ordinary and partial differential equations. We address the question of approximation of reduced-order PDF equations by multi-level coarse graining, perturbation series and operator cumulant resummation. Numerical examples are presented for stochastic resonance, stochastic advection-reaction and Burgers equations.