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Quadrilateral axisymmetric 4-node hybrid-stress elements using the Quadrilateral Area Coordinate Method (QACM)

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The Quadrilateral Area Coordinate Method (QACM) is an effective tool for developing quadrilateral elements insensitive to mesh distortions [1-3]. In this paper, two quadrilateral axisymmetric 4-node hybrid-stress elements using the QACM, AQACHQ4 and AQACHQ6, are developed based on the Hellinger-Reissner and the modified Hellinger-Reissner principles, respectively. Numerical examples show the present models are free of volumetric locking in the almost incompressible state and less sensitive to various mesh distortions.

Element formulations

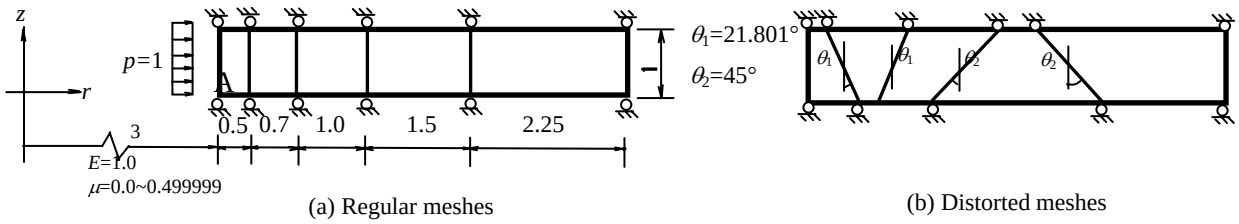
The hybrid-stress elements need both assumed displacement and stress fields. For new element AQACHQ4, its assumed displacement fields, which are expressed in terms of the QACM, are given by Reference [4]; and its stress fields are given by

$$\boldsymbol{\sigma} = \mathbf{P}\boldsymbol{\beta} = \begin{bmatrix} 1 & 0 & 0 & J_{11}^2(L_4-L_2) & J_{21}^2(L_3-L_1) & 0 & 0 \\ 0 & 1 & 0 & J_{12}^2(L_4-L_2) & J_{22}^2(L_3-L_1) & 0 & 0 \\ 0 & 0 & 1 & J_{11}J_{12}(L_4-L_2) & J_{21}J_{22}(L_3-L_1) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & J_{12}(L_3-L_1)+J_{22}(L_4-L_2) \end{bmatrix} \begin{Bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_7 \end{Bmatrix} \tag{1}$$

Then, the element formulations can be obtained by the Hellinger-Reissner principle. For the other new element AQACHQ6, the additional internal displacements and stresses expressed in terms of the QACM are considered, and the final formulations are obtained by the modified Hellinger-Reissner principle.

Numerical example: thick-walled cylinder under internal pressure

As shown in Figure 1, an infinitely long thick-walled cylinder under internal pressure is analysed by taking a slice of unit thickness [5]. The numerical results are listed in Table I.



(a) Regular meshes (b) Distorted meshes

Figure 1. Thick-walled cylinder under internal pressure

Table I. MacNeal-Harder test

		Regular meshes(a)		
μ	Axi-Q4	RHAQ6[6]	**AQACHQ4**	**AQACHQ6**
0.0	0.994	0.994	**1.000**	**0.994**
0.25	0.990	—	**1.000**	**0.991**
0.3	0.988	0.990	**1.000**	**0.990**
0.49	0.847	0.986	**1.000**	**0.986**
0.499	0.359	0.986	**1.000**	**0.986**
0.4999	0.053	0.986	**1.000**	**0.986**
0.499999	0.00056	0.986	**1.000**	**0.986**
		Distorted meshes(b)		
μ	Axi-Q4	RHAQ6[6]	**AQACHQ4**	**AQACHQ6**
0.0	0.989	0.986	**0.997**	**0.989**
0.25	0.984	—	**0.997**	**0.987**
0.3	0.982	0.987	**0.997**	**0.986**
0.49	0.816	0.983	**0.996**	**0.982**
0.499	0.315	0.983	**0.997**	**0.982**
0.4999	0.044	0.983	**0.997**	**0.982**
0.499999	0.00044	0.983	**0.997**	**0.982**

It can be seen that, in both the regular and distorted meshes, the new two elements can yield good results which are free of volumetric locking. And in particular, the element AQACHQ4 produces the best solutions. The efficiency of the QACM for developing simple, effective and reliable finite elements are again demonstrated.

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