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A COROTATIONAL TETRAHEDRAL ELEMENT FOR LARGE-DISPLACEMENT ANALYSIS OF SMA STRUCTURES

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Compared to hexahedral elements, tetrahedral elements are especially attractive in practical engineering applications involving complex geometries, since they allow for a very straightforward mesh generation, whereas automatic mesh generation is often not feasible for the former [1]. The aim of the present paper is the development of a new tetrahedral finite element accounting for material and geometric nonlinearities, in the framework of the corotational formulation.

The corotational approach is based on the idea of separating rigid body motions from purely deformational ones [2]. It is especially attractive for problems involving large displacements and small strains. In fact in those cases existing high-performance linear elements can be reused as core elements in the geometrically nonlinear context, after large rigid body motions have been filtered out. In this work a polar decomposition based corotational formulation is exploited [3, 4] and original closed-form formulas are derived for the efficient computation of the nodal residual vector and of the consistent tangent stiffness tensor.

A four-node solid tetrahedron with three translational and three rotational degrees of freedom (DOFs) per node is adopted as core element. The rationale is that the accuracy of those elements is intermediate between that of the linear and quadratic elements with translations only, yet they contribute to a much smaller bandwidth and thus solution time compared to the quadratic elements [5]. Existing four-node tetrahedrons with six DOFs per node include TET4RX [6], HT4R [5], RGNTet4 [1]. The latter exhibits a somewhat stiff behaviour, whereas the first is not frame invariant. Hence, HT4R is a natural candidate as core element. The derivation of HT4R is based on a modified Hellinger–Reissner functional, that treats the rotation and the skew symmetric stress as independent fields to formulate a stabilization scheme. Unfortunately, approaches based on Hellinger–Reissner

functional may not be efficient in a nonlinear material framework, usually involving the direct strain-stress relationship. An enhancement of HT4R is here proposed, based on a modified Hu-Washizu functional:

$$\mathcal{F}(\mathbf{u}, \varepsilon, \sigma, \omega, \tau) = \left\langle \psi(\varepsilon) - \sigma \cdot \varepsilon + \sigma \cdot \mathbf{D}\mathbf{u} + \tau \cdot (\mathbf{L}\mathbf{u} - \omega) - \frac{1}{2\gamma\mu}||\tau||^2 \right\rangle \tag{1}$$

where $< \cdot >$ is the integral operator over the element domain, ε is the strain tensor, ψ is the free energy density, σ and τ are the symmetric and skew-symmetric parts of the stress tensor, respectively, $\mathbf{u}$ is the displacement vector, ω is the independently assumed rotation vector, μ is a typical stiffness modulus, γ is an arbitrary positive non-dimensional parameter. Moreover, $\mathbf{D}$ and $\mathbf{L}$ are the differential operators for deriving, respectively, the strain and rotation from the displacement field. As in the original formulation, the corner rotations are introduced by transformation of the mid-side translational DOFs of a ten-node tetrahedron, and the same stress modes are adopted, which were reduced to minimum without sacrificing the frame invariance and proper rank of the element [5]. Here two different choices are made for the assumed strain field: either the same interpolation as the stress field is assumed, amounting to fitting the displacement-derived strain field into the assumed strain field by means of a least squares procedure, or a piece-wise constant discontinuous strain field is adopted, related to the quadrature rule.

Besides elastic case studies, simulations of NiTi stents used in biomedical engineering are presented. Stent structures are usually designed to significantly reduce their diameter during the insertion into a catheter. Thereby large rotations and displacements, usually combined with small to moderate strains, occur. Chemical and transformation-strain energy, as well as dissipation, are accordingly accounted for in formulation (1).

REFERENCES

[1] R. Tian, H. Matsubara, and G. Yagawa. Advanced 4-node tetrahedrons. *Int. J. Numer. Methods Eng.*, 68(12):1209–1231, 2006.

[2] B. Nour-Omid and C. C. Rankin. Finite rotation analysis and consistent linearization using projectors. *Comput. Meth. Appl. Mech. Eng.*, 93(3):353–384, 1991.

[3] G. F. Moita and M. A. Crisfield. A finite element formulation for 3-D continua using the co-rotational technique. *Int. J. Numer. Methods Eng.*, 39(22):3775–3792, 1996.

[4] F. Caselli and P. Bisegna. Polar decomposition based corotational framework for triangular shell elements with distributed loads. *Int. J. Numer. Methods Eng.*, 95(6):499–528, 2013.

[5] K. Y. Sze and Y. S. Pan. Hybrid stress tetrahedral elements with Allman's rotational D.O.F.s. *Int. J. Numer. Methods Eng.*, 48(7):1055–1070, 2000.

[6] T. P. Pawlak, S. M. Yunus, and R. D. Cook. Solid elements with rotational degrees of freedom: Part II–tetrahedron elements. *Int. J. Numer. Methods Eng.*, 31(3):593–610, 1991.