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APPROXIMATED LAX PAIRS FOR A ONE-DIMENSIONAL FLUID-STRUCTURE MODEL

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We propose a reduced-order technique based on the concept of Lax pairs to solve non-linear evolution Partial Differential Equations (PDE). We apply this method to a one-dimensional fluid-structure interaction model classically used to model the pulse wave propagation in a network of arteries.

In his seminal work [4], Lax proposed a formalism to integrate a class of nonlinear evolution PDEs. He introduced a pair of linear operators $\mathcal{L}(u)$ and $\mathcal{M}(u)$, where u denotes the solution of the PDE. The eigenfunctions of $\mathcal{L}(u)$, are propagated by a linear PDE involving $\mathcal{M}(u)$. For some PDEs, the eigenvalues of $\mathcal{L}(u)$ have the remarkable property to be constant in time, as soon as $\mathcal{L}(u)$ and $\mathcal{M}(u)$ satisfy the Lax equation:

$$\partial_t \mathcal{L} + \mathcal{L}\mathcal{M} - \mathcal{M}\mathcal{L} = 0 \tag{1}$$

This formalism can be applied to a wide range of problems arising in many fields of physics, such as Korteweg-de Vries (KdV), Camassa-Holm, Sine-Gordon or nonlinear Schrödinger equations.

The main idea presented in this work is to use the eigenfunctions of $\mathcal{L}(u)$ as a basis to approximate the solutions of the PDE. Contrary to standard reduced-order modeling techniques, like the Proper Orthogonal Decomposition (POD) or the Reduced Basis Method (RBM), the basis is therefore time dependent and is propagated with the operator $\mathcal{M}(u)$. This property makes the method especially well-suited to problems featuring propagation phenomena, which are known to be difficult to tackle with POD.

To define a model that is genuinely "reduced", the number of eigenfunctions used to approximate the solution has to be small. This seems to be the case for various problems of practical interest. This has been recently pointed out by Laleg, Crépeau and Sorine who used the eigenfunctions of a Schrödinger operator, playing the role of $\mathcal{L}(u)$, to provide a parsimonious representation of the arterial blood pressure [2, 3]. Their signal processing technique, called SCSA (for Semi-Classical Signal Analysis), has been our starting point and has been used in a preliminary version of the present work [1].

The operator $\mathcal{M}$ being unknown in general, we propose a way to approximate it. Then we apply the method to an hyperbolic one-dimensional system used to model fluid-structure interaction in arteries. Our approach allows to reduce the computational cost by more than one order of magnitude.

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