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Simulations of Acoustic Wave Propagation with Generalized Multiscale Finite Element Methods

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Numerical simulations of acoustic and elastic wave propagation make important contributions both to fundamental studies of waves in complex media and to methods of inversion and imaging. One of the most common approaches used for this task is the time domain finite difference algorithm, and this solution has been applied to acoustic, elastic and viscoelastic wave equations to address many different problems in geophysics [e.g., 4, 6]. However, these methods face challenges when applications require very detailed earth models, as fine discretizations greatly increase required computational time and memory. Applications to estimation of subsurface properties for petroleum exploration are an important example of such as case, because improved data acquisition now provides larger and more complete data sets that are able to constrain such models more and more reliably. Simulations therefore require increasingly finer descriptions of heterogeneity to take advantage of such data.

This challenge provides the motivation for our recent development of new multiscale finite element methods that have the goal of providing more efficient solutions for modeling wave propagation in very complex media [1, 2]. The general goal of multiscale methods is to achieve this objective by incorporating the effects of fine scale features in computations carried out on a coarse grid with reduced CPU time and memory.

We will present an enhanced generalized multiscale finite element method (GMsFEM) for modeling acoustic wave propagation in complex media. There have been several previous developments of multiscale methods [3, 5]. Our GMSFEM approach has several advantages. In particular, our scheme is designed to choose as basis functions for the coarse grid the most important modes controlling propagation on the underlying fine grid. This is an improvement over previous work using a few arbitrary bases.

The GMsFEM has the following general procedure for computing the multiscale basis functions that incorporate the effects of fine scale heterogeneity. First, a large set of generic bases are computed by solving a relevant problem on each coarse cell using the fine grid information, thus defining a “snapshot space”. This snapshot space includes bases representing coarse-grid boundary modes and a second set of interior standing modes. Given the set of boundary modes, proper orthogonal decomposition (POD) is applied to choose the most important functions. For the interior standing modes, the lowest frequency modes are selected. This combined set of basis functions is then utilized in a Galerkin-type numerical method. It is important to note that the basis function generation is done only one time for a single model, thus allowing dramatic acceleration of computation for petroleum exploration problems requiring repeated simulations to reproduce a field experiment.

We have applied this method to test models combining fine-scale stochastic heterogeneity with a general layered structure (Fig. 1). Wave propagation effects thus include both complex scattering and coherent reflections from layer boundaries. Comparisons with conventional finite-differences show that the GMsFEM solutions can be made accurate with a reasonable choice of basis functions and that computation times are reduced.

In summary, the GMsFEM applies a new and innovative approach to generating basis functions that represent the influence of fine-scale heterogeneity in multiscale simulations carried out on a coarse-scale grid. This approach can achieve reliable and accurate results by combining bases computed one time in advance of simulations. The approach therefore has strong potential for applications in modeling and imaging where many simulations need to be carried out for a single model, since each simulation will be accelerated significantly.

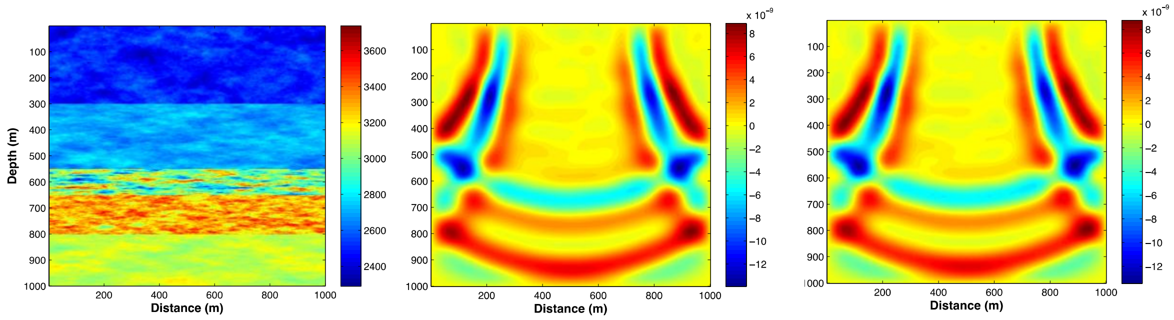

Fig. 1: Left: earth model with acoustic wave velocity in m/s; center: fine-scale reference solution at propagation time 0.4 s ; right: representative multiscale solution at time 0.4 s.

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