11th. World Congress on Computational Mechanics (WCCM XI)
5th. European Conference on Computational Mechanics (ECCM V)
6th. European Conference on Computational Fluid Dynamics (ECFD VI)
July 20 - 25, 2014, Barcelona, Spain

A simple adaptive procedure for separated representations in engineering applications

Ch. Ghnatios[1], A. Leygue[1], M. Beringhier[2], J.J. Rodenas[3], J. Fuenmayor[3], F. Chinesta[1]

[1] GeM Institute, Ecole Centrale nantes
1 rue de la Noë, F-44300 Nantes, France
chady.ghnatios@ec-nantes.fr

[2] Institut P', ENSMA
1 Avenue Clément Ader, F-86961 Futuroscope cedex, France
marianne.beringhier@ensma.fr

[3] Depto. de Ingeniería Mecánica y de Materiales, Universitat Politècnica de València
Camino de Vera, 46022 Valencia, Spain
jjrodena@mcm.upv.es

Key Words: *Separated representations, PGD, Parametric solutions, Error estimation, Adaptive procedures*

In spite of the impressive progress made during the last decades in mathematical modelling and techniques of scientific computing, many problems in science and engineering remain intractable. Among them, we can cite those related to high-dimensional models. Other challenging scenarios are those requiring many direct solutions of a given problem or those needing very fast solutions.

Over the last ten years a novel technique, called Proper Generalized Decomposition –PGD- has been intensively developed in order to address the before mentioned computational issues. The PGD builds by means of a successive enrichment strategy a numerical approximation of the unknown fields in a separated form involving "a priori" unknown functions of individual or clustered coordinates of the problem. Thus, it seems an appealing technique for addressing highly multidimensional models, circumventing the redoubtable "curse of dimensionality".

Many challenging problems can be efficiently cast into a multidimensional framework. For instance, material parameters, boundary and initial conditions, geometrical parameters defining the domain in which the model is defined, … can be regarded as extra-coordinates of the problem in addition to the usual coordinates such as space and time. In the PGD framework, this enriched model is solved only once to yield a general or parametric solution that includes all particular solutions for specific values of the parameters. Thus, optimization of complex problems, uncertainty quantification, simulation-based control and real-time simulation become feasible, even in highly-complex scenarios and deployed computing devices [1].

As just indicated PGD based approximations are based on the use of separated representations

of the different fields involved in the physical models. Imagine a model involving the scalar unknown field depending on d coordinates $u(x_1,\cdots,x_d)$. Its simplest separated representation reads:

$$u(x_1,\cdots,x_d) \approx \sum_{i=1}^{i=N} \prod_{j=1}^{j=d} F_i^j(x^j) \tag{1}$$

Such representation involves two approximations. The first concerns the finite sum, whose truncation involving N terms represents a first source of numerical error. The second error is related to the approximations of the different functions involved in (1). Thus, when using a standard finite element interpolation we approximate each function $F_i^j(x^j)$ from:

$$F_i^j(x^j) \approx \sum_{k=1}^{k=M^j} F_i^j\left(x_k^j\right) \cdot P_k(x^j) \tag{2}$$

where $F_i^j(x_k^j)$ represents the nodal values of function $F_i^j(x^j)$ at the nodal positions x_k^j and $P_k(x^j)$ a polynomial verifying the standard interpolation property $P_k(x_m^j) = \delta_{k,m}$. Obviously the finite approximation (2) represents a second error source. For improving the approximation accuracy we can increase either the number of terms of the finite sum N or the number of nodes involved in the mesh of each coordinate M^j [2].

In a previous work we proposed a strategy for estimating the error as a function of the number of terms N involved in the finite sum [3]. In this work we propose a simple adaptive procedure for adapting the different meshes associated with each coordinate, inspired of the simple adaptive procedure originally proposed by Zienkiewicz and Zhu [4].

REFERENCES

[1] F. Chinesta, A. Leygue, F. Bordeu, J.V. Aguado, E. Cueto, D. Gonzalez, I. Alfaro, A. Ammar, A. Huerta. Parametric PGD based computational vademecum for efficient design, optimization and control. Archives of Computational Methods in Engineering, 20/1, 31-59, 2013.

[2] F. Chinesta, R. Keunings, A. Leygue. The Proper Generalized Decomposition for advanced numerical simulations. A primer. Springerbriefs, Springer, 2014.

[3] A. Ammar, F. Chinesta, P. Diez, A. Huerta. An error estimator for separated representations of highly multidimensional models. Computer Methods in Applied Mechanics and Engineering, 199 1872–1880, 2010.

[4] O.C. Zienkiewicz, J.Z. Zhu. A simple error estimator and adaptive procedure for practical engineering analysis. International Journal for Numerical Methods in Engineering, 24, 337-357, 1987.