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ENCOMPASSING INCOMPRESSIBILITY AND STRAIN LOCALIZATION IN PLASTICITY WITH MIXED FE

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Tackling incompressibility and localization in plasticity with standard finite elements is known to be an unfeasible task as stress locking and mesh dependency spoil the numerical solution. Mixed finite element formulations represent a suitable alternative to mitigate these drawbacks. As it has been shown in previous works by Cervera *et al.* [1], a mixed formulation in terms of displacements and pressure not only provides a propitious solution to the problem of incompressibility, but also it was found to possess the needed robustness in case of strain concentration.

This work presents a displacement-strain and displacement-stress mixed finite element framework [2, 3] as a fundamental stress-accurate enhancement of the displacement-pressure formulation above mentioned.
When complex physics is present, higher accuracy is required to solve the field of interest. Indeed, in many industrial applications, assuring the level of precision on the prediction of the stress state of the material under analysis is pivotal in many practical cases.

First the formulation is presented to address the construction of the finite element approximation. Due to mixed nature of the problem, the selected interpolation spaces must respect the *Inf-Sup Condition*. In this work, all variables fields are interpolated with functions of the same order: it is well known that such choice does not satisfy the *Inf-Sup Condition*. Consequently, a Variational Multi Scale (VMS) approach is used to stabilize the mixed problem as presented in [4].
Incompressibility and quasi-incompressibility conditions are addressed as a natural extension of the method [5]. Moreover, global and local error norms are discussed to support the advantages of the proposed method.

Therefore, a set of benchmark problems is illustrated, using both linear-linear P1P1 and

Q1Q1 elements in 2D and 3D cases. The objective is to demonstrate how this method can effectively cope with strain localization phenomena in plasticity for both pressure sensitive and insensitive failure surfaces. In particular, Drucker-Prager and Von Mises plasticity models are analysed in detail. It is shown that the numerical solution matches the theoretical solution on angles of strain concentration bands.

Concluding, it is remarked how mixed finite element formulation could represent the key to the solution of both incompressible and localization problems in mechanics whenever it is required a consistent and highly accurate numerical model.

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