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A VARIATIONAL MULTI-SCALE METHOD WITH SPECTRAL APPROXIMATION OF THE SUB-SCALES

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This work introduces a Variational Multi-Scale (VMS) method for the numerical solution of convection-diffusion equations for which the small scales are approximated by spectral expansions. This is a kind of bubble VMS method as the small scales are assumed to vanish at interfaces between elements. On each element a family of eigenfunctions of an approximated convection-diffusion operator is constructed. Although the convection-diffusion operator is not self-adjoint, it can be approximated by normal operators, and a corresponding family of eigenfunctions can be calculated analytically for rectangular elements. This allows to locally solve in analytical terms the local small scale problems.

We prove that the constructed family of eigenfunctions spans $H_0^1(K)$ for any element K of the grid. Also, that the procedure is convergent to the solution of the convection-diffusion equations for multi-dimensional problems. We present some numerical tests that yield an excellent accuracy and satisfy the maximum principle for strongly convection-dominated problems, yet with a single eigenfunction for approximating the sub-scales.

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