11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20 - 25, 2014, Barcelona, Spain

TRANSIENT THERMOELASTIC ANALYSIS OF A FUNCTIONALLY GRADED HOLLOW SPHERE WITH PIECEWISE POWER LAW

Yoshihiro Ootao[1] and Masayuki Ishihara[2]

[1] Department of Mechanical Engineering, Graduate School of Engineering, Osaka Prefecture University, 1-1 Gakuen-cho, Naka-ku, Sakai, 599-8531 JAPAN, ootao@me.osakafu-u.ac.jp
[2] Department of Mechanical Engineering, Graduate School of Engineering, Osaka Prefecture University, 1-1 Gakuen-cho, Naka-ku, Sakai, 599-8531 JAPAN.

Key Words: *Thermoelasticity, Functionally Graded Materials, Hollow Sphere, Transient State, Piecewise Power Law*

This study is concerned with the exact analysis of thermal stresses in functionally graded materials (FGMs). Thermal stresses in various materials may reach their peaks in a transient state rather than in a steady state. Therefore, we focus on the transient problem of a FGM. Since it is difficult to obtain the exact solution of such a problem due to the nonlinearity of the governing equations, some approximate approach is required. One of the approaches is to express the material profiles by some specific functions of location. As the transient thermoelastic problems, two-dimensional solutions of the single-layered functionally graded cylindrical panels and hollow cylinders were obtained by this approach. This approach, however, has a drawback in the sense that the specific functions in the sigle layer lack the applicability to arbitrary profiles. Another approach is to apply the theory of laminated composites. The transient thermal stress problems for several models were analyzed by this approach. This approach has the drawback of discontinuity in the material profiles at interfaces.

In order to overcome such drawbacks, both approaches were integrated. Ootao et al. analyzed the transient one-dimensional thermoelastic problems in the FGM hollow cylinder [1] and FGM hollow circular disk [2] by the piecewise-power model. In the model, material properties are expressed by the power functions that are defined in each virtual layer and are continuous at all the interfaces. The model is advantageous because it can describe arbitrary profiles of material properties. Ootao and Ishihara extended the analysis by the model to the asymmetric transient thermoelastic problems in the FGM hollow cylinder [3].

To the authors' knowledge, however, the exact analysis for the transient two-dimensional thermoelastic problem of functionally graded hollow sphere by the model has not been reported.

Therefore, we analyze the transient thermoelastic problem for a functionally graded hollow sphere due to axisymmetrical surface heating by employing the piece-wise power model. The material profiles can be chosen aribitrarily. The thermal conductivity, the specific heat, the Young's modulus, and the coefficient of linear thermal expansion are assumed to be expressed as power functions of the radial coordinate, and their values are continuous at the interfaces. We assume that the sphere is initially at zero temperature and are suddenly heated axisymmetrically on the inner and outer surfaces by surrounding media. The

transient two-dimensional temperature is analyzed exactly by the method of Laplace transformation and separation of variables. Then, by solving the Navier's equations, the two-dimensional exact solution for the thermoelastic response is obtained under the boundary condition in which the inner and outer surfaces are free from traction. Numerical calculations are carried out for a functionally graded hollow sphere composed of titanium alloy and zirconium oxide. Some numerical results for the temperature change and the stress distributions are shown graphically. Furthermore, the influence of the material profiles on the temperature and thermoelastic response is investigated.

REFERENCES

[1] Y. Ootao, Transient *Thermoelastic Analysis for a Multilayered Hollow Cylinder with Piecewise Power Law Nonhomogeneity*. *J. Solid Mech. Marer. Eng.*, Vol. 4, pp. 1167–1177, 2010.

[2] Y. Ootao and Y. Tanigawa, *Transient Thermoelastic Analysis for a Multilayered Hollow Circular Disk with Piecewise Power Law Nonhomogeneity. J. Thermal Stresses,* Vol. 35, pp. 75–90, 2012.

[3] Y. Ootao and M. Ishihara, *Asymmetric transient thermal stress of a functionally graded hollow cylinder with piecewise power law, Struc. Eng. Mech,* Vol. 47, pp. 421–442, 2013.