

XFEM FOR A CRACK MODEL WITH STRIP-YIELD CRACK TIP PLASTICITY

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The extended Finite Element Method (XFEM) is one of the most popular methods for the simulation of fracture. In the vicinity of the crack tip the classical FE ansatz functions are supplemented with additional “enrichment functions” to improve the approximation of the true solution. Along the crack the elements are enriched with jump function, allowing the crack to propagate without any mesh dependencies.

While for brittle cracks it is standard to use the asymptotic solution of Williams, [3], as enrichment functions, there exist different approaches to deal with cohesive cracks. As cohesive cracks show no stress singularity at the tip, one approach, which is implemented also in some commercial FE solvers, uses jump functions for enrichment exclusively, e.g. in [5]. Another approach, which is better suited for applications where coarse meshes must be used, is based on analytic solutions satisfying a cohesive force boundary condition in the crack tip region for the enrichment. E.g. in [4] Xiao and Karihaloo derived asymptotic fields for special cohesive laws under mixed-mode conditions and used them in XFEM simulations.

This contribution uses XFEM to simulate a straight 2d cohesive cracks governed by the Dugdale strip-yield crack-tip plasticity [1]. For this model an analytic solution was derived and used in a hybrid crack tip element applying the Hellinger Reissner principle in [2]. The solution of the overall problem is a superposition of solutions for the linear elastic and the cohesive crack. The linear elastic part consists also of higher order terms and is traction free at the crack edges. The solution for the cohesive crack shows a predefined closure stress at the crack tip, such that the stress singularity of the linear elastic near field solution is removed. Within the cohesive zone the traction equals exactly the predefined closure stress, while, in contrast to the work of Xiao and Karihaloo, for the rest of the crack the faces are traction free. The derivation of the solution will be summarized in this contribution, as well as its reformulation, such that the solution can be used as crack

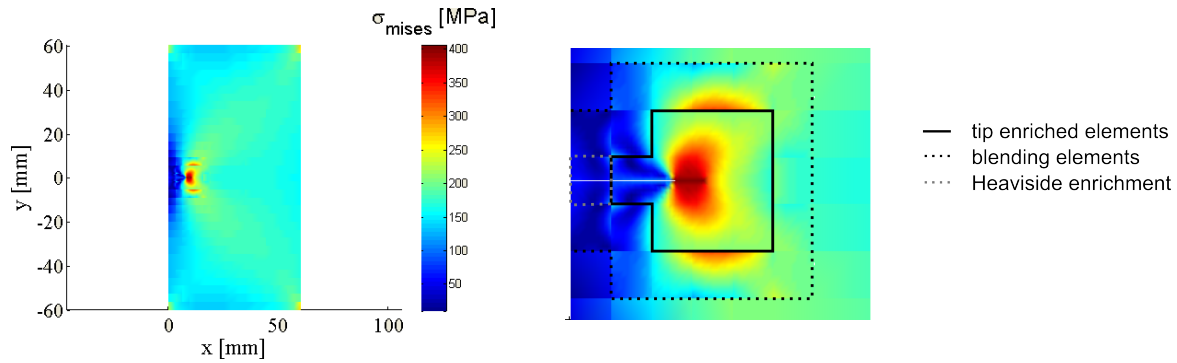


Figure 1: Von Mises stress field of a mode I tension test. Left: whole specimen. Right: close up of crack tip.

tip enrichment in the XFEM formalism. It is emphasized that in the presented approach, the physical crack tip (beginning of the cohesive zone) can grow in arbitrarily small steps through the mesh. Due to the nonlinear character of the boundary condition in the cohesive zone, the length of the zone must be determined iteratively. The iterative solution procedure will be described briefly. As the compatible stresses of the FE computation are discontinuous, within the enriched domain the stresses are projected onto the analytical solution. A projection treatment of blending elements will be developed also.

Considering an mode I test, as in Fig. 1, the results are compared against a high resolution FE computation. Results are in good agreement.

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