11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20–25, 2014, Barcelona, Spain

RECOVERY-BASED GUARANTEED UPPER ERROR BOUNDS IN ENERGY NORM OF THE FINITE ELEMENT SOLUTION FOR THE LINEAR ELASTICITY PROBLEM

E. Nadal[*1], P. Díez[2], J.J. Ródenas[1], M. Tur[1] and F.J. Fuenmayor[1]

[1]Centro de Investiación en Tecnología de Vehículos (CITV)
Universitat Politècncica de València, Camino de Vera, s/n, E-46022, Valencia, Spain
e-mail: enadalsoriano@gmail.com, {jjrodena, matuva, ffuenmay}@mcm.upv.es
[2]Departamento de Matemática Aplicada, Universitat Politècnica de Catalunya
Campus Nord UPC-C2, Jordi Girona 1, E-08034, Barcelona, Spain
e-mail: pedro.diez@upc.es

Key words: *Finite element method, error bounding, error estimation, recovery*

Nowadays, high-technology companies are increasingly demanding numerical techniques for simulations. In the linear elasticity framework, these simulations are mainly carried out by means of the Finite Element Method (FEM). The FEM is a widely extended numerical simulation tool in the linear elasticity context. It provides an approximated solution to the real behaviour of the analysed component. The FEM uses a discretization process that introduces the so-called discretization error of the Finite Element (FE) solution. In this contribution we will consider the discretization error as the only source of error. Since the very beginning of the FEM, great efforts have been done in order to evaluate the discretization error in energy norm, bringing up the three main branches for error estimation: the *residual-based* error estimators [1], dual error estimators such as the Constituve Relation Error [3] and the *recovery-based* error estimators [4]. The two first are able to provide upper bounds of the error in energy norm. On the other hand recovery-based error estimators provide accurate estimations, are robust and easy to implement but, so far, they are unable to provide error bounds.

The recovery-based error estimators are based on the use of recovered fields obtained from the FE solution which provides a solution of a higher quality. Among others recovery techniques we can highlight the Super-convergent Patch Recovery (SPR) technique introduced by Zienkiewicz and Zhu [5], the SPR-C [6] which introduces local (at patch

With the support of FP7 ITN Funding under Grant No. 289361 'INSIST', Ministerio de Economía y Competitividad of Spain (DPI2010-20542), FPU program (AP2008-01086) and Generalitat Valenciana (PROMETEO/2012/023).

level) equilibrium constraints, or the SPR-CX [7] specially adapted for the eXtended FEM framework. All these error estimators have a high accuracy.

In this work, we propose a recovery-based error estimator based on the use of the SPR-C technique that is able to provide very accurate upper error bounds in energy norm. The proposed technique, so called FUB, uses, on the one hand the ZZ error estimator and, on the other hand, an explicit (constant-dependent) upper bound of the correction terms introduced in [2] to account for the lacks of internal and boundary equilibrium of the SPR-C recovered fields, directly providing upper error bounds. Additionally, we also propose a technique to numerically evaluate the constant C for each problem. Numerical results show the high accuracy of the bounding technique, its asymptotically exactness and the robustness of the evaluation of C for each problem.

REFERENCES

[1] I. Babuška and W.C. Rheinboldt. A-posteriori error estimates for the finite element method. *Int. J. Numer. Meth. Eng.*, Vol. **12**(10), 1597–1615, 1978.

[2] P- Díez, J.J. Rdenas and O.C. Zienkiewicz. Equilibrated patch recovery error estimates: simple and accurate upper bounds of the error. *Int. J. Numer. Meth. Eng.*, Vol. **69**(10), 2075–2098, 2007.

[3] P. Ladevèze and D. Leguillon. Error estimate procedure in the finite element method and applications.. *SAIM Journal on Numerical Analysis*, Vol. **69**(10), 2075–2098, 2007.

[4] O.C. Zienkiewicz and J.Z. Zhu. A simple error estimator and adaptive procedure for practical engineering analysis. *Int. J. Numer. Meth. Eng.*, Vol. **24**(2), 337–357, 1987.

[5] O.C. Zienkiewicz and J.Z. Zhu. The superconvergent patch recovery and a posteriori error estimates. Part 1: The recovery technique. *Int. J. Numer. Meth. Eng.*, Vol. **33**(7), 1331–1364, 1992.

[6] J.J. Ródenas, M. Tur, F.J. Fuenmayor, and A. Vercher. Improvement of the superconvergent patch recovery technique by the use of constraint equations: the SPR-C technique. *Int. J. Numer. Meth. Eng.*, Vol. **70**(6), 705–727, 2007.

[7] J.J. Ródenas, O.A. Fonzález-Estrada, J.E. Tarancón and F.J. Fuenmayor. A recovery-type error estimator for the extended finite element method based on singular+smooth stress field splitting. *Int. J. Numer. Meth. Eng.*, Vol. **76**(4), 545–571, 2008.