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CONFIGURATION–DEPENDENT INTERPOLATION IN HIGHER ORDER 2D BEAM FINITE ELEMENTS

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Here we discuss interpolation functions for the field variables and their variations in relation to geometrically non–linear planar beam finite elements of Reissner's type [4] within the context of a non–standard, configuration–dependent interpolational setting.
We derive the configuration–dependent interpolation functions as an extension of the Borri and Bottasso helicoidal interpolation [1] for two–noded elements, which is independent on the choice of the beam reference axis, to higher–order elements. In linear analysis, the proposed configuration–dependent interpolation coincides with the higher–order linked interpolation [3], which is known to produce exact results for polynomial loading.
In non–linear analysis, the rotational part of the the configuration–dependent interpolation reduces to strain–invariant interpolation introduced by Jelenić and Crisfield [2].
Such a configuration–dependent interpolation for the position field of a 2D problem is given by the expression $\mathbf{r}^h = \sum_{i=1}^{N} \mathbf{J}_i \mathbf{r}_i$ with

$$\mathbf{J}_i = \delta_{Ii}\left(\mathbf{I} - \sum_{m=1}^{N} \mathbf{N}_m\right) + \mathbf{N}_i, \tag{1}$$

where $\mathbf{r}_i$ and $\mathbf{r}^h$ are the position vector of node i and the interpolated position vector, δ_{Ii} is the Kronecker symbol ($\delta_{Ii} = 1$ if $I = i$ and $\delta_{Ii} = 0$ otherwise), I is the chosen reference node, and $\mathbf{I}$ is a two–dimensional unity matrix. Function $\mathbf{N}_i$ is defined as:

$$\mathbf{N}_i = I_i \frac{\psi_i \sin\psi^h}{\psi^h \sin\psi_i}[\cos(\psi^h - \psi_i)\mathbf{I} + \sin(\psi^h - \psi_i)\ \hat{\mathbf{e}}], \tag{2}$$

where I_i is the Lagrangian polynomial of order $N-1$ (N is the number of nodes), index h represents an interpolated quantity whereas index i represents a nodal quantity. Further,

$\psi^h = \beta\ (\varphi^h - \varphi_I)/2$ and $\psi_i = \beta\ (\varphi_i - \varphi_I)/2$, with φ_i as the actual rotational nodal unknown and φ^h as the interpolated unknown rotational field. Matrix $\hat{\mathbf{e}}$ is defined as $\hat{\mathbf{e}} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$. Depending on the coefficient β two variants of the configuration–dependent interpolation are possible. Interpolation with coefficient $\beta = 1$ gives a solution independent of the beam reference axis but does not represent the exact field distribution in the linear limit. Interpolation with coefficient $\beta = 2/N$ provides the exact field distribution in the linear limit, but fails to provide the solution which is independent of the position of the beam reference axis.

In the following figure, a graphical presentation of the proposed configuration–dependent interpolation functions is given for a two–noded element. In the case when there are no local rotations, the proposed interpolation reduces to Lagrangian polynomials, here presented by a thick line.

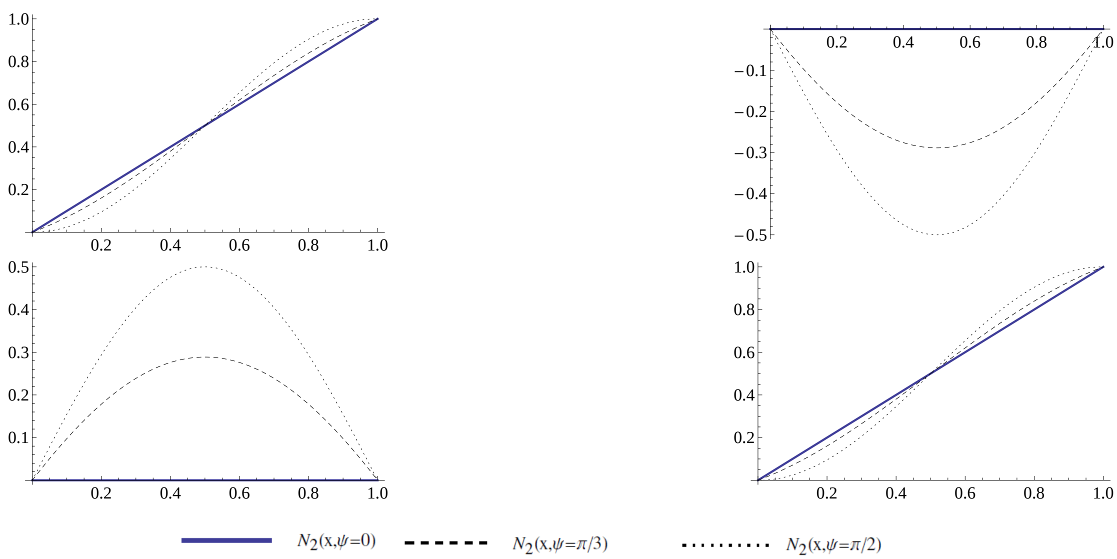

Figure 1: Function $\mathbf{N}_2(x_1, \psi_2 = 0, \dfrac{\pi}{3}, \dfrac{\pi}{2})$.

REFERENCES

[1] M. Borri and C. Bottasso. An intrinsic beam model based on a helicoidal approximation–Part I: Formulation. *Int. J. Numer. Methods Eng.*, Vol. **37**(13), 2267–2289, 1994.

[2] G. Jelenić and M.A. Crisfield. Geometrically exact 3D beam theory: Implementation of a strain–invariant finite element for statics and dynamics. *Comput. Methods Appl. Mech. Eng.*, Vol. **171**(1–2), 141–171, 1999.

[3] G. Jelenić and E. Papa. Exact solution of 3D Timoshenko beam problem using linked interpolation of arbitrary order. *Arch. Appl. Mech.*, Vol. **81**(2), 171–183, 2011.

[4] E. Reissner. On one–dimensional finite–strain beam theory: the plane problem. *Z. Angew. Math. Phys.*, Vol. **23**(5), 795–804, 1972.